

請依題號順序將題號及答案清楚寫在答案卷上 (10 題共 100 分)

1~5: 單選題: 每題恰有一解, 答對一題得五分不倒扣 (共 25 分)

1. The Laplace transform of  $t$  is : (A)  $\frac{1}{s}$  (B)  $\frac{1}{s^2}$  (C)  $\frac{1}{s+1}$  (D)  $\frac{1}{(s+1)^2}$ .

2. The Laplace transform of  $e^t$  is : (A)  $\frac{1}{(s-1)}$  (B)  $\frac{1}{(s+1)}$  (C)  $\frac{1}{(s-1)^2}$  (D)  $\frac{1}{(s+1)^2}$ .

3. The Laplace transform of  $e^t \cos t$  is : (A)  $\frac{1}{s^2+2s+2}$  (B)  $\frac{1}{s^2-2s+2}$  (C)  $\frac{s-1}{s^2+2s+2}$  (D)  $\frac{s-1}{s^2-2s+2}$ .

4. The inverse Laplace transform of  $\frac{2}{(s-3)^3}$  is : (A)  $te^t$  (B)  $2te^t$  (C)  $t^2e^t$  (D)  $t^2e^{2t}$ .

5. The inverse Laplace transform of  $\frac{1}{s^4-s^2}$  is : (A)  $\cosh t + t$  (B)  $\cosh t - t$  (C)  $\sinh t + t$  (D)  $\sinh t - t$ .

計算題: (共 75 分)

6. 求  $x^2y'' - 5xy' + 9y = 0$  之解 (10%)

7. 求  $y''' + 3y'' + 3y' + y = 30e^{-x}$ ,  $y(0) = 3$ ,  $y'(0) = -3$ ,  $y''(0) = -47$  之解 (15%)

8. Please find the general solutions of the first order partial differential equation? (16%)

$$2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = 1$$

9. Matrix  $B = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$ , please calculate (a) eigenvalues (特徵值) (b) eigenvectors (特徵向量) of the matrix? (16%)

10. Find the frequency of oscillation of a pendulum of length  $L$ , neglecting air resistance and the weight of the rod, and assuming  $\theta$  to be so small that  $\sin \theta$  practically equals  $\theta$ . (Hint: based on physical model using force balance to develop mathematical model; ordinary differential equation and solve the ODE) (18%)

