

5% for each question

1. For  $y'' + 3y' + 2y = t$ , its general solution is (1)  $y = c_1e^t + c_2e^{2t} + At$  (2)  $y = c_1e^t + c_2e^{2t} + At + B$

(3)  $y = c_1e^{-t} + c_2e^{-2t} + At$  (4)  $y = c_1e^{-t} + c_2e^{-2t} + At + B$  (5) None

2. For  $y'' + y' + y = e^t$ , its general solution is (1)  $y = e^{\frac{t}{2}}(c_1 \cos \frac{\sqrt{3}t}{2} + c_2 \sin \frac{\sqrt{3}t}{2}) + Ae^t$

(2)  $y = e^{\frac{t}{2}}(c_1 \cos \frac{\sqrt{3}t}{2} + c_2 \sin \frac{\sqrt{3}t}{2}) + Ae^t + B$  (3)  $y = e^{-\frac{t}{2}}(c_1 \cos \frac{\sqrt{3}t}{2} + c_2 \sin \frac{\sqrt{3}t}{2}) + Ae^t$

(4)  $y = e^{-\frac{t}{2}}(c_1 \cos \frac{\sqrt{3}t}{2} + c_2 \sin \frac{\sqrt{3}t}{2}) + Ae^t + B$  (5) None

3. For  $y'' + 2y' + y = \sin t$ , its general solution is (1)  $y = c_1e^{-t} + c_2te^{-t} + A \sin t + B \cos t$

(2)  $y = c_1e^{-t} + c_2e^{-t} \ln t + A \sin t$  (3)  $y = c_1e^{-t} + c_2e^{-t} \ln t + A \sin t + B \cos t$

(4)  $y = c_1e^{-t} + A \sin t + B \cos t$  (5) None

4. For  $x^2y'' + 3xy' + y = x$ , its general solution is (1)  $y = c_1x^{-1} + Ax + B$

(2)  $y = c_1x^{-1} + c_2x^{-1} \ln x + Ax + B$  (3)  $y = c_1x^{-1} + c_2x^{-1} \ln x + Ax$  (4)  $y = c_1x^{-1} + c_2x^{-1} \ln x + B$

(5) None

5. For  $x^2y'' - 2xy' + 2y = \ln^3 x$ , its general solution is (1)  $y = c_1x^{-1} + c_2x^{-2} + A \ln^3 x$

(2)  $y = c_1x + c_2x^2 + A \ln^3 x$  (3)  $y = c_1x + c_2x^2 + A \ln^3 x + B \ln^2 x$

(4)  $y = c_1x + c_2x^2 + A \ln^3 x + B \ln^2 x + C \ln x$  (5) None

6. Given  $F(s) = \int_0^\infty f(t)e^{-st} dt$  and  $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$ , then the Laplace transform of  $t^2u(t-1)$  is

(1)  $\frac{2}{s^3}$  (2)  $\frac{1}{s^3}$  (3)  $\frac{e^{-s}}{s^3}$  (4)  $\frac{e^s}{s^3}$  (5) none

7. The Laplace transform of  $e^{2t} \sin 3t$  is (1)  $\frac{3}{s^2 + 9}$  (2)  $\frac{s}{s^2 + 9}$  (3)  $\frac{3}{s^2 - 4s + 9}$  (4)  $\frac{3}{s^2 - 4s + 13}$

(5)  $\frac{s-2}{s^2 - 4s + 13}$

8. The Laplace transform of  $t^2 f(t)$  is (1)  $\frac{d^2 F(s)}{ds^2}$  (2)  $F(s)e^{2s}$  (3)  $s^2 F(s)$  (4)  $\frac{F(s)}{s^2}$  (5) none

9. For a nonsingular square matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then (1)  $A^{-1} = \frac{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}{bc - ad}$  (2)  $A^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$

(3)  $A^{-1} = \frac{\begin{bmatrix} d & b \\ c & a \end{bmatrix}}{ad - bc}$  (4)  $A^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{bc - ad}$  (5) none

10. Given  $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ , then (1)  $A^2 = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}$  (2)  $A^2 = \begin{bmatrix} 4 & 4 & 1 \\ 0 & 4 & 4 \\ 0 & 0 & 4 \end{bmatrix}$  (3)  $A^3 = \begin{bmatrix} 8 & 4 & 1 \\ 0 & 8 & 4 \\ 0 & 0 & 8 \end{bmatrix}$

(4)  $A^3 = \begin{bmatrix} 8 & 4 & 0 \\ 0 & 8 & 4 \\ 0 & 0 & 8 \end{bmatrix}$  (5) none

11. The eigenvalues of  $A = \begin{bmatrix} -1 & 1 \\ 4 & 2 \end{bmatrix}$  are : (1)  $\lambda_1 = 3, \lambda_2 = -2$  (2)  $\lambda_1 = 2, \lambda_2 = -3$  (3)  $\lambda_1 = 4, \lambda_2 = -3$

(4)  $\lambda_1 = -3, \lambda_2 = 4$  (5) none

12. The value of  $\left(\frac{1-\sqrt{3}i}{2i}\right)^6$  is : (1) 1 (2) -1 (3) 0 (4) 2 (5) None

13. The derivative of  $f(z) = 2z^3 - 2iz^2 + 6 - 3i$  at  $z_0 = 2i$  is : (1) 8 (2) -8 (3) 16 (4) -16 (5) None

14. The solution of the partial differential equation  $u_x = y^2 + \sin y$  with  $u(1, y) = y^3$  is :

(1)  $u = xy^2 + x \sin y + y^3 - y^2 - \sin y$  (2)  $u = xy^2 + y \sin x + y^3 - y^2 - \sin x$

(3)  $u = xy^2 + x \cos y + y^3 - y^2 - \cos x$  (4)  $u = xy^2 + y \cos x + y^3 - y^2 - \cos y$  (5) None

15. The interval of the function  $f(x) = 5 \sin \frac{2}{3}x$  is : (1)  $\pi$  (2)  $2\pi$  (3)  $3\pi$  (4)  $4\pi$  (5) None

16. The radius of convergence of the series  $\sum_{n=0}^{\infty} \frac{n}{5^n} x^n$  is : (1) 1 (2) 3 (3) 5 (4) 7 (5) None

17.  $\vec{A} = (2, -1, 4)$ ,  $\vec{B} = (-1, 4, 3)$ ,  $\vec{C} = (3, 1, -1)$ , which of the following is correct?

(1)  $\vec{A} \cdot (\vec{B} \times \vec{C}) = -36$  (2)  $\vec{A} \cdot (\vec{B} \times \vec{C}) = -74$  (3)  $\vec{B} \cdot (\vec{A} \times \vec{C}) = 28$  (4)  $\vec{B} \cdot (\vec{A} \times \vec{C}) = 46$

(5) None

18.  $f(x, y, z) = xy + e^{yz}$ ,  $\vec{u} = \left(\frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$ , which of the following is wrong?

(1)  $\nabla f(x, y, z) = (f_x, f_y, f_z) = (y, x + ze^{yz}, ye^{yz})$       (2)  $\nabla f(1, 0, 1) = (0, 2, 0)$

(3)  $D_u f(x, y, z) = \frac{1}{\sqrt{14}}y - \frac{2}{\sqrt{14}}(x + ze^{yz}) + \frac{3}{\sqrt{14}}ye^{yz}$       (4)  $D_u f(1, 1, 0) = \frac{2}{\sqrt{14}}$       (5) None

19.  $\vec{A} = (xz, yx, zy)$ , which one of the following is correct?

(1)  $\text{div } \vec{A} = z + x + y$       (2)  $\text{div } \vec{A} = xz + yx + zy$

(3)  $\text{curl } \vec{A} = (x, y, z)$       (4)  $\text{curl } \vec{A} = (yz, xz, xy)$       (5) None

20. About even function and odd function, which one of the following is correct?

(1)  $f(x) = x^2$  is odd function      (2)  $f(x) = x^5$  is even function

(3)  $f(x) = x \sin x$  is odd function      (4)  $f(x) = e^{x^2}$  is odd function      (5) None