1. Let A = {a, b, c, d, e}. Find the **relation** on A with the following relationship matrix (10%)

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

2. Given the set of integers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   (a) How many ways are there to arrange the integers? (5%)
   (b) How many ways are there to arrange the integers so that **the first digit is greater than 1**? (5%)
   (c) How many ways are there to arrange the integers so that **the last digit is less than 7**? (5%)
   (d) How many ways are there to arrange the integers so that **the first digit is greater than 1 and the last digit is less than 7**? (5%)

3. Find the coefficient of $a^3b^2c^3$ in the expansion of $(2a + 3b + c)^8$. (10%)

4. How many integers must be selected from the set {1, 2, 3, . . ., 200} to ensure that there are two integers m and n such that gcd(m, m) = 1. (10%)

5. Let $\mathbf{E} = [\mathbf{v}_1, \mathbf{v}_2]$ and $\mathbf{F} = [\mathbf{u}_1, \mathbf{u}_2]$, where $\mathbf{v}_1 = [5, 2]^T$, , $\mathbf{v}_2 = [7, 3]^T$, and let $\mathbf{u}_1 = [3, 2]^T$, $\mathbf{u}_2 = [1, 1]^T$
   (a) Find the **transition matrix** corresponding to the change of basis from $[\mathbf{e}_1, \mathbf{e}_2]$ to $[\mathbf{u}_1, \mathbf{u}_2]$ (10%)
   (b) Find the **transition matrix** from $[\mathbf{v}_1, \mathbf{v}_2]$ to $[\mathbf{u}_1, \mathbf{u}_2]$ (10%)

6. Let $\mathbf{A} = \begin{bmatrix} 0 & 1 & 2 & 3 \\ -2 & -2 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & -2 & -3 \end{bmatrix}$, find det(**A**) (10%)

7. Determine the *nullspace* of the following matrix (10%)

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

8. Let **A** be a nonsingular matrix. Show that $\mathbf{det(A^{-1})} = \dfrac{1}{\mathbf{det(A)}}$ (10%)