1. (12%) Consider the following relations on {1, 2, 3},

$R_1 = \{(1,1), (2,3), (3,3), (3,2)\}$,

$R_2 = \{ (2,2), (3,1), (3,3), (3,2), (1,1)\}$

$R_3 = \{(1,2), (3,2), (2,3), (3,3)\}$

$R_4 = \{(1,2), (3,2), (3,1), (2,3)\}$

$R_5 = \{(1,1), (2,2), (3,3), (3,2)\}$

$R_6 = \{(1,1), (3,3), (3,1), (1,3), (2,2)\}$

(a) Which of these relations are reflexive?

(b) Which of these relations are symmetric?

(c) Which of these relations are anti-symmetric?

(d) Which of these relations are transitive?

2. (10%) For X={1, 2, 3} , Y={a, b, c, f}

(a) The number of the relations from X to Y.

(b) How many functions f: $X \to Y$ are **one to one**?

(c) How many functions f: $Y \to X$ are **onto**?

3. (3%) Give the recursive definition for $C_n = 5n+9$, n= 0,1,2,…

4. (5%) Let $p(x)$ and $r(x)$be the following open statements:

$p(x): x \le 3$

$r(x): x+1 \text{ is odd}$

If the universe consists of all following integers, the following statements are true or false?

(a) $p(7) \vee r(7)$

(b) $\neg p(-4) \wedge \neg r(-3)$

5. (5%)Simplify the expression $\overline{\overline{(A \cup B) \cap C} \cup \overline{B}}$

6. (10%) Construct a state diagram for a finite state machine that recognizes each occurrence of "10" in a string and has length $2k$, $k \in Z^+$.

7. (5%) Define the closed binary operation $f: Z \times Z \to Z$ by

$$f(a,b) = a + 3ba + b$$

Is f commutative? (Try to prove your answer)

8. (10%)(**LU Factorization**) Factor the matrix **A** into a product a unit lower triangular matrix **L** times an upper triangular matrix **U**, where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ -2 & 2 & 7 \end{bmatrix}$$

9. (**Determinant**) Let **A** and **B** be 3×3 matrices with det(**A**) = 4 and det(**B**) = 3. Find the value of:
   (a) det(3**A**) (3%)
   (b) det($\mathbf{A}^{-1}\mathbf{B}$) (3%)

10. (10%) (**Change of Basis**) Find the **transition matrix** corresponding to the change of basis from [$\mathbf{v}_1$, $\mathbf{v}_2$] to [$\mathbf{u}_1$, $\mathbf{u}_2$],
    where $\mathbf{v}_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ and $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

11. (10%) (**Linear Transformation**) Let L be the linear transformation mapping $\boldsymbol{R}^3$ into $\boldsymbol{R}^3$ defined by

$$L(\mathbf{x}) = (2x_1 - x_2 - x_3, -x_1 + 2x_2 - x_3, -x_1 - x_2 + 2x_3)^T$$

Determine the standard matrix representation **A** of L.

12. (**Eigenvalues, Eigenvectors and Diagonalization**) Given the following matrix:

$$\mathbf{A} = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$$

   (a) Find the eigenvalues and the corresponding eigenvectors for the matrix (9%)
   (b) Factor **A** into a product of the form $\mathbf{XDX}^{-1}$, where **D** is diagonal (5%)