

1. How many different binary search trees are there for 6 keys? Justify your answer. (10%)
2. A variation of the scheme for sparse matrix representation involves representing only the nonzero terms in an one-dimensional array v in the order described. In addition, a strip of $n \times m$ bits, $bits[n][m]$ is also kept. $bits[i][j] = 0$ if $A[i][j] = 0$, and $bits[i][j]=1$ if $A[i][j] \neq 0$. The figure below illustrates an example of the representation for the sparse matrices.

$$A = \begin{pmatrix} 15 & 0 & 0 & 22 & 0 & 15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} 15 \\ 22 \\ 15 \\ 11 \\ 3 \\ -6 \\ 91 \\ 8 \end{bmatrix}$$

On a computer with 16 bits per integer, how much storage is needed to represent a sparse integer matrix $A_{n \times m}$ with t nonzero terms? (10%)

3. Given the input $\{4371, 1323, 6173, 4199, 4344, 9679, 1989\}$, a fixed table size of 10, and a hash function $H(X) = X \bmod 10$, show the results of hash table if the following methods are used for collision.
 - (a) Linear probing hash table. $(H(X)+i)$ (5%)
 - (b) Quadratic probing hash table. $(H(X)+i^2)$ (5%)
4. Suppose there is an article which is composed by the following 8 characters $\{a,b,c,d,e,f,g,h\}$. The corresponding occurrence of each character in this article is also given as follows: $\{6, 2, 10, 9, 3, 15, 8, 20\}$. Show the total length of bits needed for the article if Huffman code is used. Also show the Huffman code for each character. (10%)
5. Construct the AVL tree of the following insertion sequence: 10, 20, 30, 40, 100, 90, 80, 70, 60, 50. (10%)
6. Bob, who is a network administrator, supervises a network of computers. He is keeping log connections between the computers in the network. Each connection is bi-directional. Two computers are interconnected if they are directly connected or if they are interconnected with the same computer. Occasionally, Bob has to decide, quickly, whether two given computers are connected, directly or indirectly, according

to the log information. Write a program which based on information input from a text file counts the number of connected components. (20%)

Input and Output

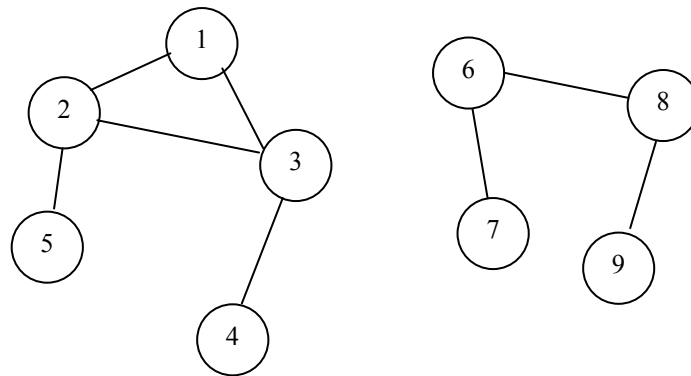
The program reads data from a text file, as follows:

1. The number of computers in the network (a strictly positive integer);
2. The number of lines which use to connect computers.
3. A list of pairs of lines.

For example, the input file illustrated in the sample below corresponds to a network of 9 computers and 8 pairs of lines.

Sample Input

```
9
8
1 2
2 3
3 1
5 2
4 3
7 6
8 6
9 8
```



Sample Output

There are 2 connected components.

7. Lucky numbers are defined as follows. Beginning with the natural numbers strike out all even ones, leaving the odd numbers 1, 3, 5, 7, 9, 11, 13, ... The second number is 3, next strike out every third number, leaving 1, 3, 7, 9, 13, ... The third number is 7, next strike out every seventh number and continue this process infinite number of times. The numbers surviving are called lucky numbers. The first few lucky numbers are:

1, 3, 7, 9, 13, 15, 21, 25, 31, 33,

Please write a program to generate all lucky numbers between 1 and 1000. (20%)

8. What is the data structure used in Heap sort? Give reasons for your answer. (10%)