1. Solving the following differential equations.

(a) (15%) $5x^2y'' - 2xy' - y = 0$, $y(x) = ?$

(b) (15%) $y'' + y' - 2y = e^{2x}$, $y(x) = ?$

(c) (15%) $y' + y = \begin{cases} 0, & 0 \le t < \pi/2 \\ \cos t, & t \ge \pi/2 \end{cases}$ .and $y(0) = 2$, $y(t) = ?$

2. (10%) $f(t) = \begin{cases} 1 & -1 \le t < 1 \\ 0 & 1 \le t < 2 \end{cases}$ and $f(t) = f(t+3)$, derive its Fourier series.

※ **Hint:** If $f(t) = f(t+T)$ then the Fourier series of $f(t)$ is $f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{2n\pi t}{T} + b_n \sin\frac{2n\pi t}{T})$

3. Matrix $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

(a) (5%) Evaluate the determinant of **A**, or $\det \mathbf{A} = ?$.
(b) (5%) Evaluate the adjoint of **A**, or $\operatorname{adj} \mathbf{A} = ?$
(c) (5%) Evaluate the inverse of **A**, or $\mathbf{A}^{-1} = ?$
(d) (10%) Find the eigenvalues and eigenvectors of **A**.

4. (10%) $\vec{F} = \left[xy^3, x^3y\right]$, R is the rectangle with vertices $(0,0)$, $(3,0)$, $(3,2)$, $(0,2)$. Please evaluate $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$ along the boundary curve C around the region R in the direction of "counterclockwise".

5. $f = x^2 + y^2 - z$, $P:(1,1,-2)$, $\vec{a} = [1,1,2]$

(a) (5%) Find ∇f at P.
(b) (5%) Find $\nabla \times \nabla f$ at P.