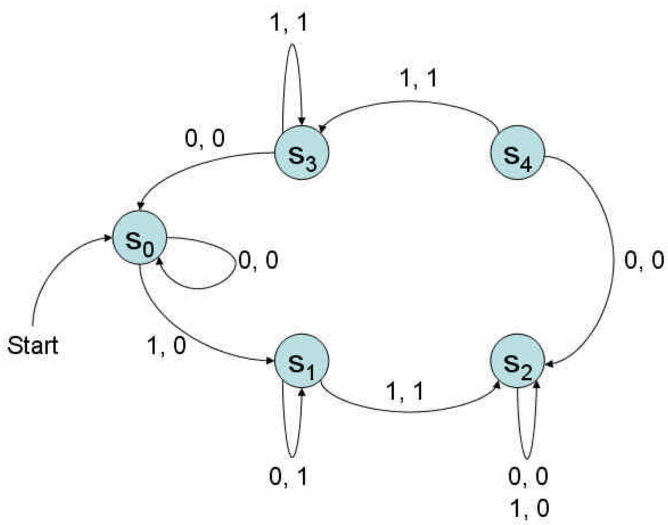
1. (10%) Simplify the following statements
   a. $\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$
   b. $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)]$
2. (10%) Write the dual statement for each of the following set-theoretic results.
   a. $U = (A \cap B) \cup (\overline{A} \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap \overline{B})$
   b. $A = A \cap (A \cup B)$
3. (10%) *Ackermann's function A(m, n)* is defined recursively for *m*, *n* ∈ **N** by

   A(0, n) = n + 1, n ³ 0

   A(m, 0) = A(m-1, 1), m > 0, and

   A(m, n) = A(m-1, A(m, n-1)), m, n > 0

   Prove that
   a. *A(1, n) = n + 2,* for all n ∈ **N**
   b. *A(2, n) = 3 + 2n,* for all n ∈ **N**
4. (10%) For each of the following relations, determine whether the relation is reflexive, symmetric, or transitive.
   a. $R \subseteq Z^+ \times Z^+$, where *a R b* if *a* divides *b*
   b. *R* is a relation on *Z*, where *x R y* if (*x* - *y*) is even.
5. (10%) A finite state machine *M = (S, I, O, v, w)* has *I = O = {0, 1}* and is determined by the following state diagram



   a. Find the state table for this machine.
   b. In which state should we start so that the input string *10010* produces the output *10000* ?

6. (10%) To visualize a three-dimensional object with plane faces (e.g., a cube), we may store the position vectors of the vertices with respect to a suitable $x_1x_2x_3$-coordinate system (and a list of the connecting edges) and then obtain a two-dimensional image on a video screen by projecting the object onto a coordinate plane, for instance, onto the x_1x_2-plane by setting $x_3=0$. To change the appearance of the image, we can impose a linear transformation on the position vectors stored. Show that a diagonal matrix **D** with main diagonal entries 3, 1, 1/2 gives from an $x=[x_j]$ the new position vector $y=\mathbf{D}x$, where $y_1=3x_1$ (stretch in the x_1-direction by a factor 3), $y_2=x_2$ (unchanged), $y_3=(1/2)x_3$ (contraction in the x_3-direction). What effect would the following matrices have in the situation described above?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q & -\sin q \\ 0 & \sin q & \cos q \end{bmatrix}, \begin{bmatrix} \cos j & 0 & -\sin j \\ 0 & 1 & 0 \\ \sin j & 0 & \cos j \end{bmatrix}, \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7. (15%) State whether the given vectors are linearly independent or dependent.
   a. [1 5 3], [2 4 6], [3 9 11]
   b. [1 0], [1 2], [3 4]
   c. [1 2 3], [4 5 6], [7 8 9]
8. (10%) Show that $(A^2)^{-1}=(A^{-1})^2$. Use this to compute $(A^2)^{-1}$ of the following matrix

$$A=\begin{bmatrix} 19 & 2 & -9 \\ -4 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

9. (15%) Let $\lambda_1, \cdots \lambda_n$, be the eigenvalues of a given matrix $A=[a_{jk}]$. Show that the matrix $k_m A^m + k_{m-1}A^{m-1} + \cdots + k_1 A + k_0 I$, which is called a polynomial matrix, has the eigenvalues $k_m \lambda_j^m + k_{m-1}\lambda_j^{m-1} + \cdots + k_1\lambda_j + k_0$ $(j=1, \cdots, n)$. The eigenvectors of that matrix are the same as those of A.