1. Find the derivative of the following functions (total 15%, 5% each):

(a) $y = \ln\sqrt{\dfrac{x+1}{x-1}}$ ; (b) $y = e^3 \ln x$ ; (c) $y = x^{x-1}$

2. Find $\lim\limits_{x \to 1} \dfrac{(\ln x)^{2005}}{x^{2005} - x^{2004}}$ (10%)

3. Find $dy/dx$ given that $y^3 + y^2 - 5y - x^2 = -4$ (10%)

4. A toy company wishes to market a new toy. When the toy is set at the price $x$, the number of unit sold is given by the equation $N(x) = x^2 - 63x + 1080$ for $0 \le x \le 50$. Find the price that will give the company maximum profit. (15%)

5. Find the following integrals. (30%)

$$(a)\int_{-1}^{3} \left| x - x^2 \right| dx, (b)\int \frac{2x^3 - 8x^2 + 9x + 1}{x^2 - 4x + 4} dx, (c)\int \sec^5 x dx.$$

6. Find the area bounded by ： $x - y^2 + 3 = 0$, $x - 2y = 0$. (10%)

7. Decide follows converge or diverge. (10%)

$$(a)\sum_{k=0}^{\infty} \frac{k^k}{k!}, (b)\sum_{k=1}^{\infty} k^3 e^{-k^4}.$$

Find the derivative of the following functions (total 15%, 5% each):

(a) $y = \ln\sqrt{\dfrac{x+1}{x-1}}$ ; (b) $y = e^3 \ln x$ ; (c) $y = x^{x-1}$

(a) $y = \dfrac{1}{2}[\ln(x+1) - \ln(x-1)]$, $\quad y' = \dfrac{1}{1-x^2}$

(b) $y' = \dfrac{e^3}{x}$

(c) $\ln y = (x-1)\ln x$, $\quad \dfrac{y'}{y} = \dfrac{x-1}{x} + \ln x$, $\quad y' = x^{x-2}(x - 1 + x\ln x)$

2. Find $\lim\limits_{x\to 1}\dfrac{(\ln x)^{2005}}{x^{2005} - x^{2004}}$ (10%)

$$\lim_{x\to 1}\frac{(\ln x)^{2005}}{x^{2005} - x^{2004}} = \lim_{x\to 1}\frac{d(\ln x)^{2005}}{d(x^{2005} - x^{2004})} = \lim_{x\to 1}\frac{2005(\ln x)^{2004}(\frac{1}{x})}{2005x^{2004} - 2004x^{2003}} = \frac{0}{1} = 0$$

3. Find *dy*/*dx* given that $y^3 + y^2 - 5y - x^2 = -4$ (10%)

$$\frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5}$$

4. A toy company wishes to market a new toy. When the toy is set at the price *x*, the number of unit sold is given by the equation $N(x) = x^2 - 63x + 1080$ for $0 \le x \le 50$. Find the price that will give the company maximum profit. (15%)

Profit $P(x) = x(N(x)) = x^3 - 63x^2 + 1080x$
$P'(x) = 3x^2 - 126x + 1080 = (x - 30)(x - 12) = 0$
$x = 30$ or $x = 12$

Check all the critical points:

| | | | | |
|---|---|---|---|---|
| *x* | 0 | 12 | 30 | 50 |
| P(*x*) | 0 | 5616 | 2700 | 21500 |

Thus, the price should be set at $50.