中　華　大　學
101 學年度碩士班招生入學考試試題紙

系所別：電機工程學系碩士班　組別：通訊、系統、電子電路、光電組、微電子暨晶片設計組　科目：工程數學　共 1 頁第 1 頁

可攜帶計算機

1. (5% each) For the matrix $A=\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$, find (a) its eigenvalues and eigenvectors (b) its diagonal matrix.

2. (5% each) Find the Laplace transform of the following functions: (a) $\sinh 2t$ (b) $e^{3t}\cos 2t$

3. (5% each) For the function $f(t)=\begin{cases}2 & 0<t<2\\ 0 & \text{else}\end{cases}$, find (a) Fourier transform (b) Fourier integral.

   [Hint: Fourier transform $F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$; Fourier integral $A(\omega)=\int_{-\infty}^{\infty} f(t)\cos\omega t dt$,

   $B(\omega)=\int_{-\infty}^{\infty} f(t)\sin\omega t dt$, and $f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos\omega t+B(\omega)\sin\omega t]d\omega$.]

4. (5% each) For the periodic function $f(t)=f(t+4)=\begin{cases}-2 & 0<t<1\\ 0 & 1\le t<4\end{cases}$, find (a) Fourier series

   $f(t)=a_0+\sum_{n=1}^{\infty}(a_n\cos n\pi t+b_n\sin n\pi t)$ (b) complex Fourier series $f(t)=\sum_{n=-\infty}^{\infty}c_n e^{jn\pi t}$.

5. (5% each) For the function $f(x,y)=x^3+y^3+z^2$ and $\nabla=\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k}$, find (a) ∇f (b) $\nabla^2 f$

6. (20%) (a) By applying the Laplace transformation technique to solve the following differential equation (b) Also reply that whether there exists a unique solution or not.

   $ty''+(4t-2)y'-4y=0,\ y(0)=1,\ y'(0)=-2$

7. (15%) Solve $y'''+y'=\csc x$

8. (15%) Solve $(\sin y\cos y+x\cos^2 y)dx+xdy=0$