

(1) (25%) The error function $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ is useful in the theory of probability. Find its Taylor polynomial so that the error is bounded by 10^{-7} for $|x| \leq b$ for a given $b > 0$, let's say $b=1$. In addition, show how to evaluate the Taylor polynomial efficiently.

(2) Please find the following answers:

(a) (5%) If $I - I_n \approx \frac{C}{n^p}$, then derive the formula $\frac{I - I_n}{I - I_{2n}} \approx 2^p$

(b) (10%) If $I - I_n \approx \frac{C}{n^p}$, derive the formula $\frac{I_{2n} - I_n}{I_{4n} - I_{2n}} \approx 2^p$

(3) (20%) In the following table of numerical integrals, and their differences, try to find the likely value of p if we assume the error behaves like $I - I_n \approx \frac{C}{n^p}$.

n	I_n	$I_n - I_{n/2}$
2	0.702877396	
4	0.781978959	0.07910
8	0.804500932	0.02252
16	0.810303086	0.005802
32	0.811764354	0.001461
64	0.812130341	0.0003660

(4) (20%) Suppose a table of values of $f(x) = \sin x$, $0 \leq x \leq 1.58$, is to be constructed, with the values of $\sin(x)$ given with a spacing of h . If linear interpolation is used in the table, how small should h be in order for the interpolation error to be less than 10^{-6}

(5) (20%) Can you state and analyze the advantages and their disadvantages of three computational schemes, Bisection method, Newton's method and Secant method.