1. (20%) Machinists who work at a tool-and-die plant must check out tools from a tool center. An average of ten machinists per hour arrive seeking tools. At present, the tool center is staffed by a clerk who is paid $120 per hour and who takes an average of 5 minutes to handle each request for tools. Since each machinist produces $200 worth of goods per hour, each hour that a machinist spends at the tool center costs the company $200. The company is deciding whether or not it is worthwhile to hire (at $ 80 per hour) a helper for the clerk. If the helper is hired, the clerk will take an average of only 4 minutes to process requests for tools. Assume that service and interarrival times are exponential. Should the helper be hired? (Give the reasons)

2. (20%) Grace lives in Taipei, but she plans to drive to Kaohsiung. Grace's funds are limited, so she has decided to spend each night on her trip at a friend's house. Grace has friends in Taoyuan, Hsinchu, Miaoli, Taichung, Changhua, Yunlin, Chiayi, Tainan. Grace knows that after one day's drive she can reach Taoyuan, Hsinchu, or Miaoli. After two days of driving, she can reach Taichung, Changhua, or Yunlin. After three days of driving, she can reach Chiayi, or Tainan. Finally, after four days of driving, she can reach Kaohsiung. To minimize the number of miles traveled, where should Grace spend each night of the trip. Please use the dynamic programming to solve the problem. (The road mileages between cities are given in Figure 1.)

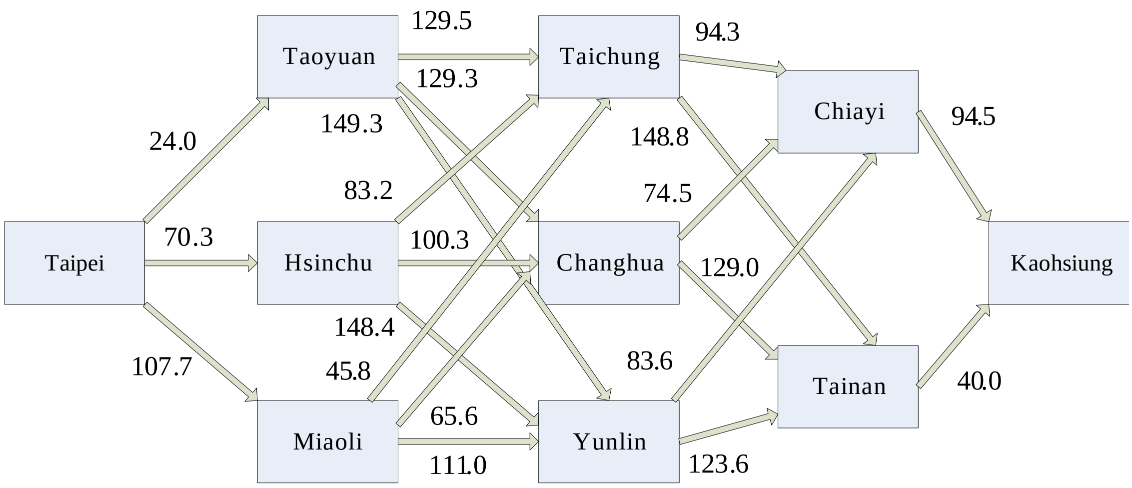

Figure 1

3. (20%) Five workers are available to perform four jobs. The time it takes each worker to perform each job is given in Table 1. The goal is to assign workers to jobs so as to minimize the total time required to perform the four jobs. (1) Formulate the linear programming. (2) Use the Hungarian method to solve the problem.

Table 1

Worker	Time (Hours)			
	Job1	Job2	Job3	Job4
1	10	15	10	15
2	12	8	20	16
3	12	9	12	18
4	6	12	15	18
5	16	12	8	12

4. (40%) Consider the following LP and its optimal tableau (Table 2).

$$\text{Max} \quad z = 8x_1 + 2x_2 + 4x_3$$
$$\text{s.t.} \quad 8x_1 + 3x_2 + x_3 \leqq 12$$
$$6x_1 + x_2 + x_3 \leqq 8$$
$$x_1, x_2, x_3 \geqq 0$$

Table 2

BV	X_1	X_2	X_3	S_1	S_2	rhs
Z	16	2	0	0	4	32
S_1	2	2	0	1	-1	4
X_3	6	1	1	0	1	8

(1) Find the dual to this LP and its optimal solution. (2) Find the range of values of the objective function coefficient of x_3 for which the current basis remains optimal. (3) Find the range of values of the objective function coefficient of x_1 for which the current basis remains optimal. (4) Find the range of the right-hand side of the second constraint for which the current basis remains optimal. (5) Find the optimal solution if we add the constraint $2x_1 + 3x_2 + x_3 \leqq 10$.