1. To find X_k and Y_k, if $X_0=0.5$, $Y_0=0.5$ (15%)

$X_{k+1}=0.1X_k+0.7Y_k$
$Y_{k+1}=0.9X_k+0.3Y_k$

Hint: $\begin{bmatrix} 0.1 & 0.7 \\ 0.9 & 0.3 \end{bmatrix}$ is a Markov matrix

2. To find the eigenvalues and eigenvectors of matrix A (15%)

$$A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

3. (A) To decide whether the u1, u2, and u3 are linear independent or linear dependent. (10%)

Where $u1=\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ $u2=\begin{bmatrix} 1 \\ -2 \\ 1 \\ -1 \end{bmatrix}$ $u3=\begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

(B) Use Gram-Schmidt algorithm to find 3 orthogonal vectors based on question 3(A). (10%)

4. For primitive statements p, q, verify that $p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology using the laws of logic. (10%)

5. Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$, where (a) $x_i \geq 0,\ \ 1 \leq i \leq 4$; (b) $x_i > 0,\ \ 1 \leq i \leq 4$ (15%)

6. Let S = {3, 7, 11 …, 115, 119}. How many elements must we select from S to ensure that there will be at least two elements whose sum is 126? State your reasoning. (10%)

7. One version of Ackermann's function A(m, n) is defined recursively for $m, n \in N$ (N is the set of nonnegative integers) by

   A(0, n) = n + 1, $n \geq 0$
   A(m, 0) = A(m-1, 1), m>0; and
   A(m, n) = A(m-1, A(m, n-1)), m, n>0
   Given that A(1, n) = n + 2 for all $n \in N$, verify that A(2, n) = 3 + 2n for all $n \in N$. (15%)