1. (10%)Five stones were chosen from diamonds, rubies, and emeralds. In how many ways can the stones be selected?

2. (15%)Let n = 420. In how many ways can one factor n as ab, where $1 < a < n$, $1 < b < n$, and gcd(a, b) = 1? (Note, the order of a, b is not relevant, e.g. 35×12 and 12×35 are counted as one way of factorization) (15%)

3. (10%)Let S = {1, 2, 3 …, 400}. Prove that if 201 integers are selected from S, then the selection must include two integers x, y where $x|y$ or $y|x$.

4. (15%)One version of Ackermann's function A(m, n) is defined recursively for $m, n \in N$ (N is the set of nonnegative integers) by

   $A(0, n) = n + 1, \quad n \geq 0$

   $A(m, 0) = A(m-1, 1)$, m>0; and

   $A(m, n) = A(m-1, A(m, n-1))$, $m, n>0$

   (a) Prove that $A(1, n) = n + 2$ for all $n \in N$ (10%)

   (b) Calculate A(2, 2) (5%)

5. (10%) Let A = $\begin{pmatrix} 2 & 0 & 0 \\ 8 & 4 & 0 \\ -2 & 0 & 6 \end{pmatrix}$, find A^{-1} = ?

6. (10%) Let A= $\begin{pmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{pmatrix}$. What is the null space of A? What is the rank of A?

7. (10%) Let A = $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$, find the eigenvalues and the corresponding eigenspaces for A ?

8. (10%) Let L: $R^2 \to R^2$ be a linear transformation. If $L((2, 1)^T) = (3, -2)^T$ and $L((-1, 1)^T) = (2, 5)^T$, determine the value of $L((5, 7)^T)$.

9. (10%) Find the best least squares fit by a linear function to the data

x	0	1	2
y	2	5	4