

1. Find the limit of the following questions: (20%)

$$(1) \lim_{x \rightarrow \infty} \frac{|x|}{|x| + 1}$$

$$(2) \lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 4x}$$

$$(3) \lim_{x \rightarrow 0^+} \frac{\ln(e^x - 1)}{\ln x}$$

$$(4) \lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x}\right)^x$$

2. Find the derivative,  $dy/dx$ , of the following questions: (20%)

$$(1) y = \frac{x^2 - 1}{x^2 + x - 2}$$

$$(2) y = 4x\sqrt{x + \sqrt{x}}$$

$$(3) y = \sin^{-1} \sqrt{1 - x^2}, 0 < x < 1$$

$$(4) y = (\sin x)^x$$

3. Evaluate the integrals of the following questions: (30%)

$$(1) \int_0^{\pi/2} \frac{3 \sin x \cos x}{\sqrt{1 + 3 \sin^2 x}} dx$$

$$(2) \int \frac{4x^3 - x^2 + 16x}{x^2 + 4} dx$$

$$(3) \int e^{2x} \cos 3x dx$$

$$(4) \int_0^{1/\sqrt{2}} 2x \sin^{-1}(x^2) dx$$

$$(5) \int \frac{x^3 dx}{\sqrt{x^2 + 4}}$$

$$(6) \int_{-\infty}^1 \frac{8x^3 dx}{(x^4 + 1)^2}$$

4. If  $y' = 6x(x + 1)(x - 2)$ , at what points, if any, does the graph of  $f$  have a local maximum, local minimum, or inflection point? On what intervals is  $f$  increasing, decreasing, concave up, or concave down? (10%)

5. The region in the first quadrant enclosed by the parabola  $y = x^2$ , the  $y$ -axis and the line  $y = 1$  is revolved about the line  $x = 3/2$  to generate a solid. Find the volume of the solid. (10%)

6. Find any local maximum, local minimum and saddle point of the function  $f(x, y) = 4xy - x^4 - y^4$ . (10%)