

請按題號順序作答，否則不予計分

一、Find the value of the limit: (20%)

$$1. \lim_{x \rightarrow \infty} e^{-x} \ln x$$

$$2. \lim_{x \rightarrow 0} \frac{x^2}{\ln(\sec x)}$$

$$3. \lim_{x \rightarrow 0} \frac{\sqrt{5x+25} - 25}{x}$$

$$4. \lim_{x \rightarrow 0} \frac{\sqrt{9x+1}}{\sqrt{x+1}}$$

二、Find the indefinite integral:

$$1. \int_1^5 \frac{x}{\sqrt{2x-1}} dx, \quad (8\%)$$

$$2. \int_1^2 x \ln x dx, \quad (8\%)$$

$$3. \int \frac{1}{1+e^x} dx, \quad (8\%)$$

三、 $f(x) = (x-3)(1-x)(x^3-x)$, find the value of $f'(1)$ (8%)

四、Find the sum of the infinite series: $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{2^{n+1}} \right)$ (8%)

五、Determine the divergence or convergence of the improper integral.

Evaluate the integral if it converges.

$$\int_0^2 \frac{1}{(x-1)^{2/3}} dx \quad (10\%)$$

六、Solve the equation (**Logistic Growth Function**, hint: to find $y=?$):

$$\frac{dy}{dt} = ky(1-y), \text{ Assume } y > 0 \text{ and } 1-y > 0 \quad (10\%)$$

七、Find the point $P(x, y, z)$ on the plane $2x+y-z-5=0$ that lies closest to the origin. (10%)

八、Show that $\lim_{\theta \rightarrow 0} \sin \theta = 0$ and $\lim_{\theta \rightarrow 0} \cos \theta = 1$ (10%)