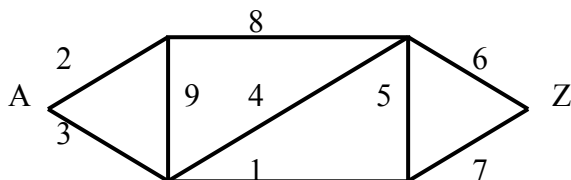1. Given the following graph. **(20pt)**
   (1) Find the minimal spanning tree. **(5pt)**
   (2) Find the shortest path from vertex A to Z. **(5pt)**
   (3) Find the minimum Hamiltonian path. **(5pt)**
   (4) Find the solution of "Traveling Salesperson Problem". **(5pt)**

2. Find A_n for the following recurrence relation. **(10pt)**

$$\sqrt{A_n} = \sqrt{A_{n-1}} + \sqrt{A_{n-2}}, \text{ where } A_0 = 1, \text{ and } A_1 = 1.$$

4. Given $|A| = 3$ and R is a binary relation on A, compute $|R|$. **(10pt)**
   (1) R is reflexive and anti-symmetric. **(5pt)**
   (2) R is the equivalence relation. **(5pt)**

4. Let F be one function, compute $|F|$ for the following properties. **(10pt)**
   (1) Given $|A| = 5$, $|B| = 3$, and F is the onto function from A to B. **(5pt)**
   (2) Given $|A| = 3$, $|B| = 5$, and F is the one-to-one function from A to B. **(5pt)**

5. Which of the following matrices are in row echelon form? Which are in reduced row echelon form? **(10pt)**

(a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

6. Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 4 & 0 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 2 & 3 & 4 \\ 0 & 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$, evaluate $\det(AB)$. **(10pt)**

7. Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$. Find the eigenvalues and the corresponding eigenspaces. **(15pt)**

8. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Write $A = LU$, where L is a lower triangular matrix and U is an

upper triangular matrix. **(15pt)**