1. Answer the **"converges"** or **"diverges"**. Assume that $a_k > 0$, $b_k > 0$, and $\sum_{k=1}^{\infty} a_k$ converges and fill in the blanks. (16%)

   (a) If $b_k \geq a_k$ for $k \geq 10$, then $\sum_{k=1}^{\infty} b_k$ _____.

   (b) If $\lim_{k \to \infty} \dfrac{a_k}{b_k} = 0$, then $\sum_{k=1}^{\infty} b_k$ _____.

   (c) If $b_k \leq a_k$ for $k \geq 6$, then $\sum_{k=1}^{\infty} b_k$ _____.

   (d) If $\lim_{k \to \infty} \dfrac{a_k}{b_k} = \infty$, then $\sum_{k=1}^{\infty} b_k$ _____.

2. Given the function $f(x) = \dfrac{1}{1-x}$, find the Taylor series about c=0 and its interval of convergence. (14%)

3. Find the derivatives of

   (a) $f(x) = 3x^4 + 5\sqrt{x}$ (5%)   (b) $f(x) = 6\sec^2 x + \dfrac{1}{\sin x}$ (5%)

4. Evaluate the following limits.

   (a) $\lim\limits_{x \to 0} \dfrac{x^2}{\cos x - x}$ (5%)   (b) $\lim\limits_{x \to 1} \dfrac{x-1}{3\ln x}$ (5%)

5. Find the following indefinite integrals

   (a) $\displaystyle\int (x^2 + x^{-2})\, dx$ (5%)   (b) $\displaystyle\int \dfrac{x+1}{\sqrt{x}}\, dx$ (5%)

   (c) $\displaystyle\int 2x(x^2 + 1)^2\, dx$ (5%)   (d) $\displaystyle\int \dfrac{x^2 + 2x + 1}{x^2 + 1}\, dx$ (5%)

   (e) $\displaystyle\int 2^{2x}\, dx$ (5%)   (f) $\displaystyle\int \dfrac{1}{1 + e^x}\, dx$ (5%)

6. Find the following integrals

(a) $\displaystyle\int_0^\infty xe^{-2x}\,dx$ (5%)   (b) $\displaystyle\int_0^1 \frac{x+3}{\sqrt{4-x^2}}\,dx$ (5%)

7. Find the area of the region between the graphs of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$ (10%)