

1-15: 單選題(75%) (每題恰有一解, 答對一題得五分, 答錯或不答時, 不倒扣也不給分)

1. What is the domain of the function $f(x) = x^{\frac{-1}{2}}$?

Select the correct answer.

- (A) $(-\infty, \infty)$ (B) $(0, \infty)$ (C) $[0, \infty)$ (D) $(-\infty, 0)$ (E) $(-\infty, 0]$

2. What is the range of the function $f(x) = x^{\frac{-1}{2}}$?

Select the correct answer.

- (A) $(-\infty, \infty)$ (B) $(0, \infty)$ (C) $[0, \infty)$ (D) $(-\infty, 0)$ (E) $(-\infty, 0]$

3. The function $f(x) = 2x^2 + 4x + 4$ is called a 2nd-order polynomial.

Which of the following statement is **not** correct?

- (A) $f(x)$ is continuous everywhere (every $x \in \mathfrak{R}$).
(B) $f(x)$ is differentiable everywhere (every $x \in \mathfrak{R}$).
(C) $f^2(x)$ is a 4th-order polynomial.
(D) $cf(x)$ is a 2nd-order polynomial for every $c \neq 0$.
(E) $[f(x)]^{\frac{1}{2}}$ is a first-order polynomial.

4. What is the rigorous definition of $\lim_{x \rightarrow 1} f(x) = 2$?

Select the correct answer.

- (A) for every $\varepsilon > 0$, there is a $\delta > 0$ such that if $0 < |x-1| < \delta$ then $|f(x) - 2| < \varepsilon$.
(B) for every $\varepsilon > 0$, there is a $\delta > 0$ such that if $|x-1| < \delta$ then $0 < |f(x) - 2| < \varepsilon$.
(C) for every $\varepsilon > 0$, there is a $\delta > 0$ such that if $0 < |x-2| < \delta$ then $|f(x) - 1| < \varepsilon$.
(D) for every $\varepsilon > 0$, there is a $\delta > 0$ such that if $|x-2| < \delta$ then $0 < |f(x) - 1| < \varepsilon$.
(E) none of above.

5. Find the limit, $\lim_{x \rightarrow 0} \frac{\cos x}{x} = ?$

Select the correct answer.

- (A) 1 (B) ∞ (C) $-\infty$ (D) does not exist (E) none of above.

6. If $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, then $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x} = ?$

Select the correct answer.

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) $\frac{2}{3}$ (E) none of above.

7. $\frac{d}{dx}(\sin x \cos x) = ?$

Select the correct answer.

(A) $\sin^2 x$ (B) $\cos^2 x$ (C) $\sin 2x$ (D) $\cos 2x$ (E) none of above.

8. $\frac{d}{dx} \sin(\sin x) = ?$

Select the correct answer.

(A) $(\sin x) \cdot \cos(\sin x)$ (B) $(\cos x) \cdot \cos(\sin x)$ (C) $(\sin x) \cdot \sin(\cos x)$
(D) $(\cos x) \cdot \sin(\cos x)$ (E) none of above.

9. $\frac{d}{dx} \left(\frac{x}{x^2 + 1} \right) = ?$

Select the correct answer.

(A) $\frac{1}{2x}$ (B) $\frac{x^2 - 1}{x^2 + 1}$ (C) $\frac{x^2 - 1}{(x^2 + 1)^2}$ (D) $\frac{1 - x^2}{(x^2 + 1)^2}$ (E) none of above.

10. Find the minimum value of $f(x) = x^3 - 3x$ on $0 \leq x \leq 2$.

Select the correct answer.

(A) 0 (B) 1 (C) 2 (D) -2 (E) none of above.

11. $\int_0^1 x^4 dx = ?$

Select the correct answer.

(A) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n}$ (B) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^4}$ (C) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^5}$ (D) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^5}{n^4}$ (E) none of above.

12. $\int_0^\pi \cos 2x dx = ?$

Select the correct answer.

(A) 0 (B) 1 (C) -1 (D) π (E) none of above.

13. $\int_0^{\infty} xe^{-x} dx = ?$

Select the correct answer.

- (A) 0 (B) 1 (C) 2 (D) e (E) none of above.

14. $\int_1^4 |x - 3| dx = ?$

Select the correct answer.

- (A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $-\frac{3}{2}$ (E) none of above.

15. $\int_{-1}^1 \frac{\tan x}{1 + x^2 + x^4} dx = ?$

Select the correct answer.

- (A) 0 (B) 1 (C) -1 (D) π (E) none of above.

16-20: 複選題(25%) (每題有多解, 完全答對一題得五分, 部份答對或不答時, 不倒扣也不給分)

16. Let $f(x)$ and $g(x)$ be two polynomial functions.

Which of the following statements are correct? Select the correct statements.

- (A) $f(x) + g(x)$ is a polynomial.
(B) $f(x) - g(x)$ is a polynomial.
(C) $f(x) \cdot g(x)$ is a polynomial.
(D) $\frac{f(x)}{g(x)}$ is a polynomial.
(E) $f(g(x))$ is a polynomial.

17. Let $f(x)$ and $g(x)$ be two functions with $f(x) \geq 1$ and $g(x) \geq 0$ for every $x \in \mathfrak{R}$.

Which of the following statements are correct? Select the correct statements.

- (A) $f(x) + g(x) \geq 1$
(B) $f(x) - g(x) \geq 1$
(C) $f(x) \cdot g(x) \geq 1$
(D) $\frac{f(x)}{g(x)} \geq 1$
(E) $f(g(x)) \geq 1$

18. Let $f(x)$ be a continuous function on \mathfrak{R} .

Which of the following statements are correct? Select the correct statements.

(A) $\int_a^a f(x)dx = 0$.

(B) $\int_a^b f(x)dx = -\int_b^a f(x)dx$.

(C) $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$ for $a < c < b$.

(D) $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$ for $a < b < c$.

(E) $\int_a^b |f(x)|dx = \left| \int_a^b f(x)dx \right|$.

19. Let $f(x)$ and $g(x)$ be two continuous functions for every $x \in \mathfrak{R}$.

Which of the following statements are correct? Select the correct statements.

(A) $f(x) + g(x)$ is continuous for every $x \in \mathfrak{R}$.

(B) $f(x) \cdot g(x)$ is continuous for every $x \in \mathfrak{R}$.

(C) $\frac{f(x)}{g(x)}$ is continuous for every $x \in \mathfrak{R}$.

(D) $f(g(x))$ is continuous for every $x \in \mathfrak{R}$.

(E) $|f(x)|$ is continuous for every $x \in \mathfrak{R}$.

20. Let $f(x)$ and $g(x)$ be two differentiable functions for every $x \in \mathfrak{R}$.

Which of the following statements are correct? Select the correct statements.

(A) $f(x) + g(x)$ is differentiable for every $x \in \mathfrak{R}$.

(B) $f(x) \cdot g(x)$ is differentiable for every $x \in \mathfrak{R}$.

(C) $\frac{f(x)}{g(x)}$ is differentiable for every $x \in \mathfrak{R}$.

(D) $f(g(x))$ is differentiable for every $x \in \mathfrak{R}$.

(E) $|f(x)|$ is differentiable for every $x \in \mathfrak{R}$.

ANSWER

1	2	3	4	5	6	7	8	9	10
B	B	E	A	D	C	D	B	D	D

11	12	13	14	15
C	A	B	C	A

16	17	18	19	20
A, B, C, E	A, E	A, B, C, D	A, B, D, E	A, B, D