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Stochastic Nonlinear Robust Adaptive Control Based on T-S Fuzzy Models

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***Abstract*—In the field of adaptive fuzzy control, there has been a severe deficiency by assuming the premise variables will usually stay within the universe of discourse in the derivation of stability of the adaptive control system. To overcome this deficiency, we develop a switching adaptive control scheme using only essential qualitative information of the plant to attain asymptotical stability of the adaptive control system for a typical first-order nonlinear system without imposing the mentioned severe assumption. The switching adaptive control system consists of an adaptive VSS controller for coarse control, an adaptive fuzzy controller for fine control, and a hysteresis switching mechanism. An adaptive VSS control scheme is proposed to force the state to enter the universe of discourse in finite time. While the premise variable is within the universe of discourse, an adaptive fuzzy control is proposed to learn the capability to stabilize the plant. At the boundary of the universe of discourse, a hysteresis switching scheme between the two controllers will be proposed. We show that after finite times of switching, the premise variables of the fuzzy system will remain within the universe of discourse and stability of the closed-loop system can be attained by applying Lyapunov direct method.**

In the current year, we focus on robust adaptive control for deterministic nonlinear systems. Based on the developed results, we shall attack the same problem for nonlinear stochstic systems in the next year.

Index Terms: Adaptive fuzzy control, switching control, T-S fuzzy model

I. Introduction

There are many deterministic fuzzy adaptive control systems which are proposed in the literature since 2000. Generally speaking, the main difficulty for adaptive fuzzy control systems arises from system uncertainty and disturbances. In the presence of these two uncertain terms, the first problem is how to guarantee uniform boundedness of parameter estimates, and the second one is how to design adaptive control law so as to guarantee system stability. In [1], it is assumed that the uncertainty term, which is also a function of plant input and system states, has an known upper bound to design a stabilizing control law. However, this assumption is unreasonable due to the following two problems.

P1. First, it is unreasonable to impose an upper bound of uncertainty term since plant input and system states may diverge before guaranteeing system stability. Especially, the upper bound is hard to know in an adaptive control scenario.

P2. Second, we can not guarantee that the premise variables will be confined in a compact universe of discourse so that the uniform approximation property holds in the analysis of the stability of the adaptive fuzzy control system.

In [2], where an adaptive control of time delay nonlinear systems is considered, problems P1 and P2 also occurred. The same situation also took place in [3] and [4]. In [5], [6], and [7], problem P1 is avoided, but problem P2 is also not considered in the analysis of the closed-loop system stability.

In [8], fuzzy systems are introduced to approximate system nonlinear functions and Lyapunov-based design techniques are employed to design stabilizing adaptive controllers to attain asymptotical stability of the state and the boundedness of the estimated parameters for regulation control problem. In their adaptive fuzzy control schemes, an essential deficiency is that the universe of discourse should depends on unknown system parameters, which is hard to define in advance. Basically, problem P2 is also not overcome in this literature.

Based on the literature survey discussed above, in this study, we shall construct a robust fuzzy adaptive control for nonlinear affine systems to overcome problems P1 and P2. We shall only use minimum information about modeling error of system uncertain terms, because adaptive controller should have the ability to learn the information of the modeling error by itself. We shall not assume that the trajectory of premise variables is limited to the universe of discourse of the fuzzy system. Without this assumption, it will be more difficult to design an stabilizing adaptive controller.

To attain our goals, we shall develop a switching adaptive control scheme to attain stability of the adaptive control system for a typical first-order nonlinear system. We shall only make some essential qualitative assumptions of the plant, instead of requiring some quantitative information of the plant, to construct an adaptive controller. The proposed switching adaptive control system consists of an adaptive VSS controller for coarse control, an adaptive fuzzy controller for fine control, and a hysteresis switching mechanism for switching of the previous two controllers. The adaptive VSS controller is used to force the premise variable to enter the universe of discourse in finite time. While the premise variable is kept within the universe of discourse, the adaptive fuzzy controller will tune its parameters and gradually learn the capability to stabilize the plant. At the boundary of the universe of discourse, a hysteresis switching scheme between the adaptive VSS control law and the adaptive fuzzy control law will be proposed. We shall show that after finite times of switching, the premise variable of the fuzzy system will remain in the universe of discourse and stability of the adaptive control system will be attained by applying the Lyapunov direct method.

The remainder of this work is organized as follows. The problem to be attacked and the hysteresis switching adaptive control scheme are described in Section 2. The adaptive VSS controller is proposed and analyzed in Section 3. Then, the considered adaptive fuzzy control is presented in Section 4. Analysis of the switching control system is made in Section 5 together with a simulation example. Finally, conclusions and discussions are given in Section 6.

Notations

For a vector $x = [\ x_1 \quad x_2 \quad \cdots \quad x_n\ ]^T$, the associated swap operation is defined as

$$\mathrm{swap}(x) = [\ x_n \quad x_{n-1} \quad \cdots \quad x_1\ ]^T$$

For a vector x, we write $x \geq 0$ if every entry of x is greater than or equal to zero.

II. Problem Formulation and The Hysteresis Switching Adaptive Control

Consider the plant

$$\dot{x} = f(x) + u \tag{1}$$

where $f(x)$ is a scalar nonlinear continuous function of the scalar variable x and $u \in R^1$ is the input. For the nonlinear continuous function $f(x)$, we make the following assumptions.

Assumption 1: $f(x)$ is a continuous function and admits its maximum $f_{\max}$ on the compact connected set Ω_x with

$$f_{\max} = \max_{x \in \Omega_x} |f(x)| \tag{2}$$

where $f_{\max}$ is an unknown positive number.

Assumption 2: The function $f(x)$ satisfies

$$\left| \frac{df(x)}{dx} \right| \leq \kappa_f \tag{3}$$

for $x \in \Omega_x$ where κ_f is an unknown positive number.

Assumption 3: For $x \notin \Omega_x$, there is a least upper bound $\psi(x)$ of $f(x)$ satisfying

$$|f(x)| \leq c_1^* |x| + c_2^* |x|^2 = \psi(x) \text{ for } x \notin \Omega_x \tag{4}$$

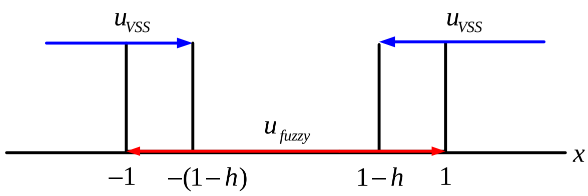

Fig. 1. Illustration of the hysteresis switching control.

where c_1^* and c_2^* are unknown positive parameters.

Assumption 4: We assume that

$$\begin{cases} xf(x) > 0, & \text{if } x \neq 0 \\ f(x) = 0, & \text{if } x = 0 \end{cases} \tag{5}$$

and $f(x)$ is a convex function for $x \in \Omega_x$. Also to simplify system analysis, we shall assume $f(x)$ is an odd function, i.e.,

$$f(-x) = -f(x) \tag{6}$$

An example of such a function $f(x)$ is given by

$$f(x) = \epsilon^* x\,|x| + \mu^* x \tag{7}$$

with $\epsilon^* > 0$ and $\mu^* > 0$ under which the equilibrium point 0 of the system dynamics (1) is unstable. In this study, we shall consider the case that the nonlinear function $f(x)$ is unknown and a fuzzy approximator $F(x|\theta)$ will be used to approximate an ideal nonadaptive stabilizing controller in the universe of discourse $\Omega_x = [-1, 1]$ where x is the only premise variable. Basically, when the state trajectory $x(t)$ is outside the universe of discourse Ω_x, by utilizing the structure information of $f(x)$ given in (4) in Assumption 3, we shall develop an adaptive VSS control $u_{VSS}(t)$ to force the state trajectory entering Ω_x. On the other hand, if the state trajectory $x(t)$ is staying within Ω_x, an adaptive fuzzy control $u_{fuzzy}(t)$ will be applied to further ensure that the system will be ultimately asymptotically stable. Since switching between these two control laws with infinite frequency at the boundary of the region Ω_x may happen, we shall use a hysteresis switching control as described in the following to avoid this problem. Let h, with $0 < h < 1$, be the hysteresis size and define the hysteresis zone Ω_h as $\Omega_h = \{x | \ 1 - h \leq |x| \leq 1\}$. The hysteresis switching control structure, as shown in Fig. 1, is described as follows. At $t = 0$, the control structure is defined as

$$u(0) = \begin{cases} u_{VSS}(0), & \text{if } |x(0)| > 1 - h \\ u_{fuzzy}(0), & \text{if } |x(0)| \leq 1 - h \end{cases} \tag{8}$$

For $t > 0$, while $x(t)$ is outside the hysteresis zone Ω_h, the control input $u(t)$ is defined as

$$u(t) = \begin{cases} u_{VSS}(t), & \text{if } |x(t)| > 1 \\ u_{fuzzy}(t), & \text{if } |x(t)| < 1 - h \end{cases} \tag{9}$$

and on the contrary, while $x(t)$ is inside the hysteresis zone Ω_h, $u(t)$ is defined as

$$u(t) = \begin{cases} u_{VSS}(t), & \text{if } u(t_-) = u_{VSS}(t_-) \\ u_{fuzzy}(t), & \text{if } u(t_-) = u_{fuzzy}(t_-) \end{cases} \tag{10}$$

We note that while applying the adaptive VSS control law u_{VSS}, the tuning parameters in the adaptive fuzzy controller will be kept invariant. On the other hand, while applying the adaptive fuzzy control law u_{fuzzy}, the tuning parameters in the adaptive VSS controller will be frozen.

The problem to be attacked is formulated as follows. For the plant in (1) under assumptions **Assumption** 1-**Assumption** 4, we shall construct an adaptive VSS controller and an adaptive fuzzy controller together with the above hysteresis switching mechanism so that the tuning parameters in the two adaptive controllers are bounded and $x(t) \to 0$ as $t \to \infty$.

III. Design and analysis of the adaptive VSS control

In this section, an adaptive VSS control will be proposed and the system behavior will be analyzed. Recall that the system function $f(x)$ has an least upper bound $\psi(x)$ with the structural information as indicated in (4) for $x \notin \Omega_x$. Here, we shall develop an adaptive VSS control $u_{VSS}(t)$ to force the state trajectory entering Ω_x when the state trajectory is outside the region Ω_x. To attain this goal, we shall construct estimates $\hat{c}_1$ and $\hat{c}_2$ of c_1^* and c_2^*, respectively, so that the following inequality

$$|f(x)| \leq \hat{c}_1 |x| + \hat{c}_2 |x|^2 \text{ for } x \notin \Omega_x$$

can be attained. Based on the estimates $\hat{c}_1$ and $\hat{c}_2$, the proposed adaptive VSS control law will be defined as

$$u_{VSS} = -(\hat{c}_1 |x| + \hat{c}_2 |x|^2 + r|x|)\text{sign}(x) \quad (11)$$

where r is a given positive constant. The tuning laws of $\hat{c}_1$ and $\hat{c}_2$ are given as

$$\begin{aligned} \dot{\hat{c}}_1 &= \Gamma_1 |x|^2, \ \hat{c}_1(0) = 0 \quad (12) \\ \dot{\hat{c}}_2 &= \Gamma_1 |x|^3, \ \hat{c}_2(0) = 0 \quad (13) \end{aligned}$$

To analyze the system response when applying the adaptive VSS control law defined in (11), (12), and (13), we consider the Lyapunov function candidate

$$V_a = \frac{1}{2}x^2 + \frac{1}{2}\Gamma_1^{-1}\tilde{c}_1^2 + \frac{1}{2}\Gamma_1^{-1}\tilde{c}_2^2$$

where

$$\begin{aligned} \tilde{c}_1 &= \hat{c}_1 - c_1^* \\ \tilde{c}_2 &= \hat{c}_2 - c_2^* \end{aligned}$$

The following lemma, adopted from [8], is required for further analysis.

Lemma 1: If $V(t,x)$ is positive definite and $\dot{V} \leq -k_1 V + k_2$ where $k_1 > 0$ and $k_2 \geq 0$ are bounded constants, then

$$V(t,x) \leq \frac{k_2}{k_1} + (V(0) - \frac{k_2}{k_1})e^{-k_1 t}$$

for all t. Also it is obvious that

$$\lim_{t\to\infty} V(t,x) \leq \frac{k_2}{k_1}$$

Lemma 2: Consider the adaptive VSS control system defined by (1), (11), (12), and (13). The trajectories of $x(t)$, $\hat{c}_1(t)$, and $\hat{c}_2(t)$ are bounded over the time interval (t_0, ∞) where t_0 is an arbitrary initial time, and $x(t)$ converges to the origin. Moreover, there is a finite time t_1 such that $x(t_1) = 1-h$ if $x(t_0) > 1-h$ or $x(t_1) = -(1-h)$ if $x(t_0) < -(1-h)$ where t_1 is a time instant with $t_1 \leq t_0 + T_1$ and

$$T_1 = \frac{V_a(t_0)}{r(1-h)^2} \quad (14)$$

Proof: The time derivative of V_a along the system trajectory of the adaptive VSS control system can be evaluated as

$$\begin{aligned} & \dot{V}_a \\ = \ & x[f(x) + u_{VSS}] + \tilde{c}_1 |x|^2 + \tilde{c}_2 |x|^3 \\ = \ & xf(x) - \psi(x)|x| + (c_1^*|x| + c_2^*|x|^2)|x| \\ & -(\hat{c}_1|x| + \hat{c}_2|x|^2 + r|x|)|x| + \tilde{c}_1|x|^2 + \tilde{c}_2|x|^3 \\ \leq \ & -r|x|^2 \leq 0 \quad (15) \end{aligned}$$

The above inequality implies that the trajectories of $x(t)$, $\hat{c}_1(t)$, and $\hat{c}_2(t)$ are bounded over the time interval (t_0, ∞) and $V_a(t)$ is a non-increasing function of t. From (12) and (13), it is obvious that both $\hat{c}_1(t)$ and $\hat{c}_2(t)$ are non-decreasing functions of t. Therefore $\hat{c}_1(t)$ and $\hat{c}_2(t)$ both converge to some finite values as $t \to \infty$. On the other hand, we have

$$\ddot{\hat{c}}_1 = 2\Gamma_1 x\left[f(x) - (\hat{c}_1|x| + \hat{c}_2|x|^2 + r|x|)\text{sign}(x)\right]$$

which is bounded, and we have $\dot{\hat{c}}_1$ is uniformly continuous. Then, Barbalat's lemma [9], we have

$$\lim_{t\to\infty} \dot{\hat{c}}_1(t) = 0$$

Consequentially, with $\dot{\hat{c}}_1 = \Gamma_1|x|^2$ defined in (12), we can conclude

$$\lim_{t\to\infty} x(t) = 0$$

From (15), there exists a positive function $Z(t) \geq 0$ such that

$$\dot{V}_a + Z = -r|x|^2$$

Integrating the last differential equation, one can get

$$\begin{aligned} V_a(t) &= V_a(t_0) + \int_{t_0}^{t} [-Z(\tau) - r|x|^2(\tau)]d\tau \\ &= V_a(t_0) - r\int_{t_0}^{t} |x|^2(\tau)d\tau - \int_{t_0}^{t} Z(\tau)d\tau \end{aligned}$$

which implies

$$1/2x^2 \leq V_a(t) \leq V_a(t_0) - r\int_{t_0}^{t} |x|^2(\tau)d\tau \quad (16)$$

Now define a function $y(t)$ as

$$y(t) = \int_{t_0}^{t} |x|^2(\tau)d\tau$$

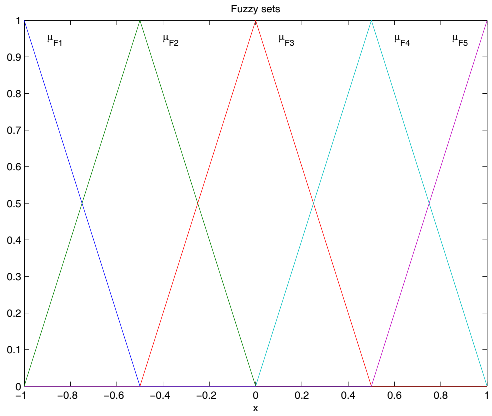

Fig. 2. A typical case of the fuzzy sets in the rule base.

which is equivalent to the following differential equation

$$dy/dt = x^2(t),\ y(t_0) = 0$$

Then, from (16), we have

$$\dot{y} + 2ry \leq 2V_a(t_0)$$

By Lemma 1, we can conclude that

$$\begin{aligned} y(t) &\leq 2V_a(t_0)\int_{t_0}^{t} e^{-2r(t-\tau)}d\tau \\ &= (\frac{1-e^{-2r(t-t_0)}}{r})V_a(t_0) \end{aligned}$$

and thus

$$\int_{t_0}^{t} x^2(\tau)d\tau \leq (\frac{1-e^{-2r(t-t_0)}}{r})V_a(t_0) < \frac{V_a(t_0)}{r} \tag{17}$$

If $x(t_0) > 1-h$, we can show that there is a finite time t_1 such that $x(t_1) = 1-h$ with $t_1 - t_0 \leq T_1$ where T_1 is defined in (14). By contradiction, assume that $x(t) > 1-h > 0$ for $t \in [t_0, t_0+T_1]$. Then, we have

$$\int_{t_0}^{t_0+T_1} x^2(\tau)d\tau > (1-h)^2 T_1 = \frac{V_a(t_0)}{r}$$

which contradicts inequality (17). This verifies the assertion. Similarly, if $x(t_0) < -(1-h)$, there is a finite time t_1 such that $x(t_1) = -(1-h)$ with $t_1 - t_0 \leq T_1$. This ends the proof. ■

IV. Design of the Adaptive Fuzzy Control

For the plant in (1), the only premise variable of the fuzzy system is x and the universe of discourse Ω_x is chosen as $\Omega_x = [-1, 1]$. The rule base of the T-S fuzzy system is defined as: for $1 \leq l \leq L$,

$$\text{Rule } l : \text{If } x \text{ is } F_l, \text{then } y = \theta_l.$$

where F_l is the fuzzy set with membership function $\mu_{F_l}(x)$ and θ_l is the value specified in the antecedent part of the $l-$th rule. The number L, which is the total number of rules, will be chosen as an odd number. A typical case is shown in Fig. 2 where the set of IF-THEN rules is complete, consistent, and continuous [10]. Based on the above rule base, the T-S fuzzy system, consisting of the singleton fuzzyifier, the product inference engine, and the center average defuzzifier [10], can be expressed as

$$F(x,\theta) = \xi^T(x)\theta \tag{18}$$

where

$$\begin{aligned} \theta &= [\theta_1, ..., \theta_L]^T, \\ \xi_l(x) &= \frac{\mu_{F_l}(x)}{\sum_i^L \mu_{F_l}(x)}, \\ \xi(x) &= [\xi_1(x), ..., \xi_L(x)]^T \end{aligned} \tag{19}$$

From Fig. 2, we can observe that

$$\sum_i^L \mu_{F_l}(x) = 1,$$

for any $x \in \Omega_x$ and

$$\xi(x) = \left[\mu_{F_1}(x), ..., \mu_{F_L}(x)\right]^T \tag{20}$$

with $L = 5$. From the triangular membership functions shown in Fig. 2, we have, for any $x \in \Omega_x$,

$$\|\xi(x)\|^2 = \mu_{F_i}^2(x) + \mu_{F_{i+1}}^2(x) \tag{21}$$

for $1 \leq i \leq L-1$ and

$$\mu_{F_i}(x) + \mu_{F_{i+1}}(x) = 1 \tag{22}$$

From (21) and (22), it is obvious that

$$\frac{1}{2} \leq \|\xi(x)\|^2 \leq 1 \tag{23}$$

Note that since $\Omega_x = [-1, 1]$ is symmetric with respect to the origin, the rule base will be chosen to symmetric in the sense that

$$\xi(-x) = \bar{\xi}(x) \tag{24}$$

where $\bar{\xi}(x) =$swap$(\xi(x))$.

Now let A_i, for $1 \leq i \leq L$, be the support of the membership function $\mu_{F_i}(x)$, i.e.,

$$A_i = \left\{x \in \Omega_x \left| \mu_{F_i}(x) > 0 \right.\right\}$$

Denote d_i as the center of the membership function $\mu_{F_i}(x)$ for $1 \leq i \leq L$ and γ_i as the point such that $\mu_{F_i}(\gamma_i) = \mu_{F_{i+1}}(\gamma_i)$ for $1 \leq i \leq L-1$. For the convenience of further analysis, now partition the universe of discourse Ω_x as $\Omega_x = \cup_{i=1}^{2L-2}\Omega_{x,i}$ where

$$\begin{aligned} \Omega_{x,2i-1} &= [d_i, \gamma_i),\ \text{for } 1 \leq i \leq \frac{L-1}{2} \\ \Omega_{x,2i} &= [\gamma_i, d_{i+1}),\ \text{for } 1 \leq i \leq \frac{L-1}{2} - 1 \\ \Omega_{x,2i-1} &= (d_i, \gamma_i],\ \text{for } \frac{L+3}{2} \leq i \leq L-1 \\ \Omega_{x,2i} &= (\gamma_i, d_{i+1}],\ \text{for } \frac{L+1}{2} \leq i \leq L-1 \end{aligned}$$

and

$$\begin{aligned}\Omega_{x,L-1} &= [\gamma_{\frac{L-1}{2}}, d_{\frac{L+1}{2}}) \\ \Omega_{x,L} &= [d_{\frac{L+1}{2}}, \gamma_{\frac{L+1}{2}}]\end{aligned}$$

We make a final note that the fuzzy system $F(x,\theta)$ in (18) admits a linear approximator structure with respect to the parameter vector θ and

$$F(x,\theta_1) - F(x,\theta_2) = \xi^T(x)(\theta_1 - \theta_2) \tag{25}$$

For $x \in \Omega_x$, we can approximate the system function $f(x)$ by the fuzzy system $F(x,\theta) = \xi^T(x)\theta$ so that

$$\min_\theta \|f(x) - F(x,\theta)\|_\infty = W$$

for some $W > 0$ due to the universal approximation property of the constructed fuzzy system [10] and the infinite norm is defined as

$$\|g(x)\|_\infty = \sup_{x\in\Omega_x} |g(x)|$$

Let's denote a best fitted parameter θ^* as

$$\theta^* \in \arg\min_\theta \|f(x) - F(x,\theta)\|_\infty$$

For $x \in \Omega_x$, we then have

$$|f(x) - F(x,\theta^*)| \le W \tag{26}$$

Finally, with respect to the membership functions shown in Fig. 2, the hysteresis size h defined in (9) will be chosen such that

$$0 < h \le \frac{1}{4} - \varepsilon_h \tag{27}$$

where ε_h is a small positive constant.

With the above definitin of the fuzzy system, the adaptive controller is defined as

$$\begin{aligned}\dot{\hat{\theta}} &= \Gamma_2 \xi x & (28) \\ u(t) &= -\hat{\theta}^T \xi(x) & (29)\end{aligned}$$

V. A Hysteresis Switching Robust Fuzzy Adaptive Control

Based on the adaptive VSS controller and the adaptive fuzzy controller, we shall study the proposed hysteresis switching robust adaptive control defined as in (9) and (10). While applying adaptive VSS controller, the closed-loop dynamics is given

$$\begin{cases} \dot{x} = f(x) - (\hat{c}_1|x| + \hat{c}_2|x|^2 + r|x|)\text{sign}(x), \\ \dot{\hat{c}}_1 = \Gamma_1 |x|^2,\ c_1(0) = 0, \\ \dot{\hat{c}}_2 = \Gamma_1 |x|^3,\ c_2(0) = 0 \\ \dot{\hat{\theta}} = 0, \end{cases} \tag{30}$$

On the other hand, by letting $\Gamma_2 = I$ in (28), the closed-loop dynamics adaptive fuzzy control system is

$$\begin{cases} \dot{x} = f(x) - \hat{\theta}^T\xi(x) \\ \dot{\hat{\theta}} = \xi x,\ \hat{\theta}(0) = 0 \\ \dot{\hat{c}}_1 = 0 \\ \dot{\hat{c}}_1 = 0 \end{cases} \tag{31}$$

Note that the value of the fuzzy approximator is given by

$$F(x|\hat{\theta}) = \hat{\theta}^T\xi(x)$$

According to the tuning law of $\hat{\theta}$ defined in (**??**) and the definition of the vector $\xi(x)$ in (20), some further properties of $\hat{\theta}$ can be discovered.

Lemma 3: Due to the structure of the fuzzy system and the tuning law of $\hat{\theta}(t)$ defined in (31), we have the following results. (i) If $A_i \subset [0,1]$, then $\hat{\theta}_i(t) \ge 0$ and $\hat{\theta}_i(t)$ is a monotone increasing function of time. On the other hand, if $A_i \subset [-1,0]$, then $\hat{\theta}_i(t) \le 0$ and $\hat{\theta}_i(t)$ is a monotone decreasing function of time. (ii) For $A_i \subset [0,1]$ or $A_i \subset [-1,0]$, if there is time t_0 such that $x(t_0) \in A_i$, then $\hat{\theta}_i(t) > 0$ for $t \ge t_0$. Similarly, if there is time t_0 such that $x(t_0) \in A_i$, then $\hat{\theta}_i(t) < 0$ for $t \ge t_0$. (iii) If $x(t) \in [1-h,1]$, then $\hat{\theta}^T(t)\xi(x(t)) \ge 0$. On the other hand, if $x(t) \in [-1,-(1-h)]$, then $\hat{\theta}^T(t)\xi(x(t)) \le 0$.

Proof: (i) Denote $\overline{A}_i$ be the closure of A_i. If $x \in \overline{A}_i \cap \overline{A}_{i+1} \subset [0,1]$, only $\hat{\theta}_i$ and $\hat{\theta}_{i+1}$ will be updated according to

$$\begin{aligned}\dot{\hat{\theta}}_i &= \mu_{F_i}(x)x \ge 0 & (32) \\ \dot{\hat{\theta}}_{i+1} &= \mu_{F_{i+1}}(x)x \ge 0 & (33)\end{aligned}$$

and $\hat{\theta}_j$ will be kept fixed for $j \ne i$ and $j \ne i+1$. On the contrary, if $x \in \overline{A}_i \cap \overline{A}_{i+1} \subset [-1,0]$, only $\hat{\theta}_i$ and $\hat{\theta}_{i+1}$ will be updated according to

$$\begin{aligned}\dot{\hat{\theta}}_i &= \mu_{F_i}(x)x \le 0 & (34) \\ \dot{\hat{\theta}}_{i+1} &= \mu_{F_{i+1}}(x)x \le 0 & (35)\end{aligned}$$

and $\hat{\theta}_j$ will be kept fixed for $j \ne i$ and $j \ne i+1$. Similarly, Since the initial guest of $\hat{\theta}$ is chosen as $\hat{\theta}(0) = 0$, equations (34)-(33) imply that if $A_i \subset [0,1]$ ($A_i \subset [-1,0]$), then $\hat{\theta}_i \ge 0$ and $\hat{\theta}_i(t)$ is monotone increasing ($\hat{\theta}_i(t) \le 0$ and $\hat{\theta}_i(t)$ is monotone decreasing). In summary, if $A_i \subset [-1,0]$ or $A_i \subset [0,1]$, then $\left|\hat{\theta}_i(t)\right|$ is monotone increasing.

(ii) If there is a time t_0 such that $x(t_0) \in A_i \subset [0,1]$, then, due to the continuity of the trajectory while using the adaptive control law, there is an interval (t_a, t_b) with $t_0 \in (t_a, t_b)$ such that $x(t) > 0$ and $\mu_{F_i}(x(t)) > 0$ for $t_0 \in (t_a, t_b)$. By (32), we have $\dot{\hat{\theta}}_i(t) > 0$ for $t_0 \in (t_a, t_b)$ and thus $\hat{\theta}_i(t) > 0$ for $t \ge t_0$. Proof of the similar case for $A_i \subset [-1,0]$ is omitted.

(iii) If $x(t) \in [1-h,1]$, then $x(t) \in \overline{A}_4 \cap \overline{A}_5 \subset [0,1]$ and

$$\hat{\theta}^T(t)\xi(x(t)) = \mu_{F_4}(x(t))\hat{\theta}_4(t) + \mu_{F_5}(x)\hat{\theta}_5(t) \ge 0$$

On the other hand, $x(t) \in [-1,-(1-h)]$, then $x(t) \in \overline{A}_1 \cap \overline{A}_2 \subset [-1,0]$ and

$$\hat{\theta}^T(t)\xi(x(t)) = \mu_{F_1}(x(t))\hat{\theta}_1(t) + \mu_{F_2}(x)\hat{\theta}_2(t) \le 0$$

This completes the proof. ■

Remark 1: Suppose that the membership functions are specified as shown in Fig. 2. Then, according to Lemma 3, we have both $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ are of non-positive values and monotone decreasing. On the other hand, $\hat{\theta}_4(t)$ and $\hat{\theta}_5(t)$ are of non-negative values and monotone increasing.

Lemma 4: The response $x(t)$ of the hysteresis switching robust adaptive control defined as in (9), (10), (**??**), and (**??**) is symmetric in the sense that if $\left\{x(t), \hat{\theta}(t)\right\}$ and $\left\{y(t), \check{\theta}(t)\right\}$ are the system responses corresponding to the initial states $x(0)$ and $-x(0)$, respectively, then $y(t) = -x(t)$ and $\check{\theta}(t) = -\bar{\theta}(t)$ where $\bar{\theta}(t)$ =swap($\hat{\theta}(t)$).

Proof : (i) First, we shall show that the response of the adaptive VSS control system is symmetric. Since both $f(x)$ and $(\hat{c}_1 |x| + \hat{c}_2 |x|^2 + r |x|)\text{sign}(x)$ are odd functions of x, so is their sum. Let $y(t) = -x(t)$. Then, by multiplying -1 to both sides of the first equation in (30), we have

$$\begin{aligned} -\dot{x} &= -f(x) + (\hat{c}_1 |x| + \hat{c}_2 |x|^2 + r |x|)\text{sign}(x) \\ &= f(-x) - (\hat{c}_1 |x| + \hat{c}_2 |x|^2 + r |x|)\text{sign}(-x) \end{aligned}$$

where we have used the property that $f(x)$ is an odd function, i.e., $f(-x) = -f(x)$. The above differential equation implies that $y(t) = -x(t)$ is the solution to the closed-loop dynamics defined as, with $y(0) = -x(0)$

$$\left\{ \begin{array}{l} \dot{y} = f(y) - (\hat{c}_1 |y| + \hat{c}_2 |y|^2 + r |y|)\text{sign}(y) \\ \dot{c}_1 = \Gamma_1 |x|^2, \ c_1(0) = 0 \\ \dot{c}_2 = \Gamma_1 |x|^3, \ c_2(0) = 0 \end{array} \right. \tag{36}$$

Note that the trajectories $c_1(t)$ and $c_2(t)$ are the same for the systems in (30) and (36).

(ii) Next, we shall show that the response of the adaptive fuzzy system defined in (31) is symmetric. Now define $\bar{\theta}$ =swap($\hat{\theta}$) and $\bar{\xi}$ =swap(ξ). Then it follows that

$$\bar{\theta}^T \bar{\xi}(x) = \hat{\theta}^T \xi(x)$$

Note that due to the symmetric structure of the fuzzy sets in the rule base, we have

$$\bar{\xi}(x) = \xi(-x)$$

Let $y(t) = -x(t)$ and $\check{\theta} = -\bar{\theta}$. Then, by multiplying -1 to both sides of the first equation in (31), we get

$$\begin{aligned} -\dot{x} &= -f(x) + \hat{\theta}^T \xi(x) \\ &= f(-x) + \hat{\theta}^T \xi(x) \end{aligned}$$

which implies

$$\begin{aligned} \dot{y} &= f(y) + \bar{\theta}^T \bar{\xi}(x) \\ &= f(y) + \bar{\theta}^T \xi(-x) \\ &= f(y) - \check{\theta}^T \xi(y) \end{aligned}$$

Similarly, by the swapping operation and the symmetric property of the vector $\xi(x)$ in (24), the second equation in (31) can be expressed as

$$\dot{\bar{\theta}} = \bar{\xi}(x)(x) = \xi(-x)(x)$$

Now multiplying -1 to both sides of the last equation, one has

$$-\dot{\bar{\theta}} = \xi(-x)(-x) = \xi(y)(y)$$

and by the definition of the vector $\check{\theta}$, it can be concluded that

$$\dot{\check{\theta}} = \xi(y)(y)$$

Therefore, the responses $y(t) = -x(t)$ and $\check{\theta} = -\bar{\theta}$ consist of the solution to the closed-loop dynamics of the adaptive fuzzy control system in (31)

$$\left\{ \begin{array}{l} \dot{y} = f(y) - \check{\theta}^T \xi(y), \ y(0) = -x(0) \\ \dot{\check{\theta}} = \xi(y)(y), \ \check{\theta}(0) = 0 \end{array} \right. \tag{37}$$

(iii) Finally, we note that the switching mechanism defined in (9) and (10) is symmetric to the origin $x = 0$.

By combining the results in parts (i), (ii), and (iii), the assertion can be concluded. This completes the proof. ■

Due to the symmetry of the responses of the switching control system as described in Lemma 4, we shall assume $x(0) > 0$ in the analysis of the dynamics of the switching control system. If $x(0) > 1 - h$, then the adaptive VSS control law in (30) will ensure that there is a finite time t_1 such that $x(t_1) = 1 - h$ and $x(t) > 1 - h$ for $t \in [0, t_1)$. At $t = t_1$, the adaptive fuzzy control law in (31) will then be applied.

A. *Analysis of switching behavior*

In this section, we shall focus on discuss switching behavior of the switching control law at the boundaries of the hysteresis zone $\Omega_h = [1-h, 1] \cup [-1, -(1-h)]$. For further analysis, we shall need some definitions.

Definition 1: We say that *continuous switching of N times at the positive boundary* $x = 1 - h$ happens at $t = t_i$ for $1 \le i \le N$ with $t_i < t_{i+1}$ if there are finite time instants t_{N+1} and $\bar{t}_i$ with $t_i < \bar{t}_i < t_{i+1}$ for $1 \le i \le N$ such that (i) the adaptive VSS controller is applied in (t_0, t_1) for some $t_0 < t_1$, (ii) the adaptive fuzzy controller is used for $t \in [t_i, \bar{t}_i]$ with $x(t) \in [1-h, 1]$ for $1 \le i \le N$, (iii) the adaptive VSS controller is applied within the interval $(\bar{t}_i, t_{i+1})$ for $1 \le i \le N$, and (iv) the adaptive fuzzy control law is used after $t = t_{N+1}$ such that there is no time instant $\bar{t}_{N+1}$ such that $\{x(t) \,|t_{N+1} \le t \le \bar{t}_{N+1}\} \subset [1-h, 1]$, $x(\bar{t}_{N+1}) = 1$, and the adaptive VSS control law is applied after $t = \bar{t}_{N+1}$. For the above situation, we also say that continuous switching of N times at the positive boundary $x = 1 - h$ happens since $t = t_1$. If $N = 1$, we say a switching at the positive boundary $x = 1 - h$ happens at $t = t_1$. Similarly, we may let $N \to \infty$, in this case we shall say continuous switching of infinite times at the positive boundary $x = 1 - h$ happens at $t = t_i$ for $1 \le i < \infty$ with

$t_i < t_{i+1}$ or continuous switching of infinite times at the positive boundary $x = 1 - h$ happens since $t = t_1$.

By the switching mechanism defined in (9) and (10), if a continuous switching of N times at the positive boundary $x = 1 - h$ happens at $t = t_i$ for $1 \le i \le N$, then we should have $x(t_i) = 1 - h$, $x(\bar{t}_i) = 1$, and $x(t) \in [1 - h, 1]$ for $t \in [t_i, \bar{t}_i]$ for $1 \le i \le N$.

Definition 2: We say that continuous switching of N times at the positive boundary $x = -(1 - h)$ happens at $t = t_i$ for $1 \le i \le N$ with $t_i < t_{i+1}$ if there are finite time instants t_{N+1} and $\bar{t}_i$ with $t_i < \bar{t}_i < t_{i+1}$ for $1 \le i \le N$ such that (i) the adaptive VSS control is applied in (t_0, t_1) for some $t_0 < t_1$, (ii) the adaptive fuzzy control law is used for $t \in [t_i, \bar{t}_i]$ with $x(t) \in [-1, -(1 - h)]$ for $1 \le i \le N$, (iii) the adaptive VSS control law is applied within the interval $(\bar{t}_i, t_{i+1})$ for $1 \le i \le N$, and (iv) the adaptive fuzzy control law is used after $t = t_{N+1}$ such that there is no time instant $\bar{t}_{N+1}$ such that $\{x(t)\,|t_{N+1} \le t \le \bar{t}_{N+1}\} \subset [-1, -(1 - h)]$, $x(\bar{t}_{N+1}) = -1$, and the adaptive VSS control law is applied after $t = \bar{t}_{N+1}$. For the above situation, we also say that continuous switching of N times at the positive boundary $x = -(1 - h)$ happens since $t = t_1$. If $N = 1$, we say a switching at the positive boundary $x = -(1 - h)$ happens at $t = t_1$. Similarly, we may let $N \to \infty$, in this case we shall say continuous switching of infinite times at the positive boundary $x = -(1 - h)$ happens at $t = t_i$ for $1 \le i < \infty$ with $t_i < t_{i+1}$ or continuous switching of infinite times at the positive boundary $x = -(1 - h)$ happens since $t = t_1$.

Definition 3: We say that a switch at the positive boundary $x = 1 - h$ (at the negative boundary $x = -(1 - h)$) happens N times at $t = t_i$ for $1 \le i \le N$ with $t_i < t_{i+1}$ if a switching at the positive boundary $x = 1 - h$ (or at the negative boundary $x = -(1 - h)$) happens at $t = t_i$ for $1 \le i \le N$. Similarly, as $N \to \infty$, we shall say a switch at the positive boundary $x = 1 - h$ (or at the negative boundary $x = -(1 - h)$) happens infinite times at $t = t_i$ for $1 \le i < \infty$ with $t_i < t_{i+1}$ or a switch at the positive boundary $x = 1 - h$ (or at the negative boundary $x = -(1 - h)$) happens infinite times since $t = t_1$.

Definition 4: We say that there is no switching at the boundary $x = 1 - h$ (or at the boundary $x = -(1 - h)$) happened since $t = t_1$ if (i) the adaptive VSS control is applied in (t_0, t_1) for some $t_0 < t_1$, (ii) the adaptive fuzzy control law is applied after $t = t_1$, and (iii) there is not a switching at the boundary $x = 1 - h$ (or at the boundary $x = -(1 - h)$) happens at $t = t_1'$ fro any $t_1' \ge t_1$.

Since the system response is symmetric as stated in Lemma 4, we shall focus on analyzing the switching property at the boundary $x = 1 - h$.

Lemma 5: Assume that there is no switching happened at the boundary $x = 1 - h$ since $t = t_1$. Then it is impossible that $\{x(t)\,|t \ge t_1\} \subset [1 - h, 1]$.

Proof: By contradiction, assume that we have $x(t_1) = 1 - h$ and the adaptive fuzzy control law in (31) is applied for $t \ge t_1$ and $\{x(t)\,|t \ge t_1\} \subset [1 - h, 1]$. From Lemma 3, we have $\hat{\theta}_4(t_1) \ge 0$ and $\hat{\theta}_5(t_1) \ge 0$. Note also that for $x(t) \in [1 - h, 1]$, we have $0 \le \mu_{F_4}(x(t)) \le \frac{1}{2}$ and $\frac{1}{2} \le \mu_{F_5}(1 - h) \le \mu_{F_5}(x(t)) \le 1$. Then, by the tuning law of $\hat{\theta}$, we have

$$\hat{\theta}_4(t) = \hat{\theta}_4(t_1) + \int_{t_1}^{t} \mu_{F_4}(x(\tau))x(\tau)d\tau \ge 0 \tag{38}$$

and

$$\begin{aligned} \hat{\theta}_5(t) &= \hat{\theta}_5(t_1) + \int_{t_1}^{t} \mu_{F_5}(x(\tau))x(\tau)d\tau \\ &\ge (t - t_1)\,\mu_{F_5}(1 - h)(1 - h) \end{aligned} \tag{39}$$

With (38) and (39), the value of $\hat{\theta}^T(t)\xi(x(t))$ can be evaluated as

$$\begin{aligned} \hat{\theta}^T(t)\xi(x(t)) &= \hat{\theta}_4(t)\mu_{F_4}(x(t)) + \hat{\theta}_5(t)\mu_{F_5}(x(t)) \\ &\ge (t - t_1)\,(1 - h)\mu_{F_5}^2(1 - h) \end{aligned}$$

Now define a constant t_M to satisfy

$$t_M > \frac{2 f_{\max}}{(1 - h)\mu_{F_5}^2(1 - h)} \tag{40}$$

Then from the first equation of the closed-loop dynamics of the adaptive fuzzy system in (31), we have

$$\begin{aligned} & x(t_1 + t_M) \\ \le\ & 1 - h + f_{\max} t_M - \frac{1}{2}(1 - h)\mu_{F_5}^2(1 - h)t_M^2 \end{aligned}$$

By the definition of t_M given in (40), we lead to a contradiction that

$$x(t_1 + t_M) < 1 - h$$

Therefore, the assumed situation is impossible. ■

In the following, we shall first investigate the learning capability of the adaptive tuning law when a switching at the boundary happens.

Lemma 6: Suppose that a switching at the boundary $x = 1 - h$ happens at $t = t_1$ and the adaptive fuzzy control is applied during the interval $[t_1, \bar{t}_1]$. Let t_2 be the next time when the adaptive fuzzy control law is applied. Then, the time difference $\bar{t}_1 - t_1$ can be estimated as

$$\bar{t}_1 - t_1 \ge \frac{h}{f_{\max}} \tag{41}$$

Moreover, we have

$$\begin{aligned} & F(x(t_2), \hat{\theta}(t_2)) \\ \ge\ & F(x(t_1), \hat{\theta}(t_1)) + \Delta_F \end{aligned} \tag{42}$$

where

$$\Delta_F = \frac{1}{2}\frac{h(1 - h)}{f_{\max}}$$

Similarly, if a switching at the boundary $x = -(1 - h)$ happens at $t = t_1$, then we have

$$F(x(t_2), \hat{\theta}(t_2)) \le F(x(t_1), \hat{\theta}(t_1)) - \Delta_F \tag{43}$$

Proof: Since a switching at the boundary $x = 1 - h$ happens at $t = t_1$, by Definition 1, we have $x(t_2) = x(t_1) = 1 - h$, $x(\bar{t}_1) = 1$, $x(t) \in [1-h, 1]$ for $t \in [t_1, \bar{t}_1]$, and the adaptive control law is applied during the interval $[t_1, \bar{t}_1]$. From the switching control law defined in (30) and (31), we see that the parameter vector $\hat{\theta}(t)$ is frozen during the interval $(\bar{t}_1, t_2)$ and thus according to the second equation of (31), we have

$$\hat{\theta}(t_2) = \hat{\theta}(\bar{t}_1) = \hat{\theta}(t_1) + \int_{t_1}^{\bar{t}_1} \xi(x(\tau))x(\tau)d(\tau) \quad (44)$$

The value of the fuzzy approximator $F(x(t), \hat{\theta}(t))$ evaluated at $t = t_2$ is given by

$$F(x(t_2), \hat{\theta}(t_2)) = \xi^T(x(t_2))\hat{\theta}(t_2) = \xi^T(x(t_1))\hat{\theta}(\bar{t}_1)$$

Now, by applying (44), we get

$$\begin{aligned} & F(x(t_2), \hat{\theta}(t_2)) \\ = \; & \xi^T(x(t_1)) \left[\hat{\theta}(t_1) + \int_{t_1}^{\bar{t}_1} \xi(x(\tau))x(\tau)d(\tau) \right] \\ = \; & F(x(t_1), \hat{\theta}(t_1)) \\ & + \int_{t_1}^{\bar{t}_1} \xi^T(t_1) \left[\xi(x(\tau)) - \xi(x(t_1)) \right] x(\tau)d(\tau) \\ & + \xi^T(x(t_1))\xi(x(t_1)) \int_{t_1}^{\bar{t}_1} x(\tau)d(\tau) \end{aligned} \quad (45)$$

Since $x(t) \in [1-h, 1]$ for $t \in [t_1, \bar{t}_1]$, from the membership functions defined in Fig. 2, we have

$$\xi(x) = \begin{bmatrix} 0 & 0 & 0 & 2-2x & 2x-1 \end{bmatrix}^T$$

and

$$\begin{aligned} & \xi(x(\tau)) - \xi(x(t_1)) \\ = \; & \begin{bmatrix} 0 & 0 & 0 & -2 & 2 \end{bmatrix}^T (x(\tau) - x(t_1)) \end{aligned}$$

for $t \in [t_1, \bar{t}_1]$. Note that $x(t_1) = 1 - h$. Therefore, it follows from inequality (27) that

$$\begin{aligned} & \xi^T(x(t_1)) \left[\xi(x(\tau)) - \xi(x(t_1)) \right] \\ = \; & \left[8(1-h) - 6 \right] (x(\tau) - x(t_1)) \\ \geq \; & 0 \end{aligned}$$

where the fact $x(\tau) - x(t_1) = x(\tau) - (1-h) \geq 0$ for $t \in [t_1, \bar{t}_1]$ has been used. Using (45) and (23), we can obtain

$$\begin{aligned} & F(x(t_2), \hat{\theta}(t_2)) \\ \geq \; & F(x(t_1), \hat{\theta}(t_1)) + \xi^T(x(t_1))\xi(x(t_1)) \int_{t_1}^{\bar{t}_1} x(\tau)d(\tau) \\ \geq \; & F(x(t_1), \hat{\theta}(t_1)) + \frac{1}{2}(1-h)(\bar{t}_1 - t_1) \end{aligned}$$

Since $x(t) \in [1-h, 1]$ for $t \in [t_1, \bar{t}_1]$, following from Lemma 3, we have

$$\dot{x}(t) = f(x(t)) - \hat{\theta}^T(t)\xi(x(t)) \leq f_{\max}$$

Therefore, with $x(t_1) = 1 - h$ and $x(\bar{t}_1) = 1$, we have

$$\bar{t}_1 - t_1 \geq \frac{h}{f_{\max}}$$

This shows inequality (42). Analysis of the case $x = -(1-h)$ is omitted. ■

Remark 2: We shall note that when the adaptive control law is applied beginning from $t = t_1$, we have

$$\begin{aligned} \dot{x}(t_1) &= f(x(t_1)) - \hat{\theta}^T(t_1)\xi(x(t_1)) \\ &= f(1-h) - \hat{\theta}^T(t_1)\xi(x(t_1)) \end{aligned}$$

Therefore, if $f(1-h)$ is greatly larger than $\hat{\theta}^T(t_1)\xi(x(t_1))$, then a switch of control law at $x = 1 - h$ may happens due to instability of the adaptive control system under the current parameter setting. To avoid continuous switching behavior at $x = 1 - h$, the only way is to increase the value of $F(x(t), \hat{\theta}(t)) = \hat{\theta}^T(t)\xi(x(t))$. Lemma 6 shows that if the adaptive control law is applied beginning from $t = t_2$ just after a switch of control law at $x = 1-h$ happened at $t = t_1$, then with the learning capability of $\hat{\theta}^T(t)$, $\dot{x}(t_1)$ will be decreased by an amount Δ_F as indicated in (42) so that the adaptive control system at $t = t_2$ has a better chance to avoid continuous switching at the boundary $x = 1 - h$.

Remark 3: Since we shall spend at least $\frac{h}{f_{\max}}$ time length to complete a switching, this implies that if continuous switching of infinite times at the positive boundary $x = 1 - h$ happens since $t = t_1$, then such a infinite-times switching can not be completed in a finite interval.

Lemma 7: Suppose that a switching at the boundary $x = 1 - h$ happens at $t = t_1$ and the adaptive fuzzy control is applied during the interval $[t_1, \bar{t}_1]$. In this case, $\hat{\theta}_5(t) - \hat{\theta}_4(t)$ is a monotone increasing function of t in the intervals $[t_1, \bar{t}_1]$. Particularly, at $t = \bar{t}_1$, we have

$$\hat{\theta}_5(\bar{t}_1) - \hat{\theta}_4(\bar{t}_1) \geq \hat{\theta}_5(t_1) - \hat{\theta}_4(t_1) + \frac{4\varepsilon_h(1-h)h}{f_{\max}} \quad (46)$$

Moreover, if $\hat{\theta}_5(t_1) \geq \hat{\theta}_4(t_1)$, then $\hat{\theta}_5(t) \geq \hat{\theta}_4(t)$ for $t \in [t_1, \bar{t}_1]$ and

$$F(\xi(x(t)), \hat{\theta}(t)) \geq F(\xi(x(t_1)), \hat{\theta}(t_1) \quad (47)$$

for $t \in [t_1, \bar{t}_1]$. In addition, if $x(t)$ is also non-decreasing, then $F(\xi(x(t)), \hat{\theta}(t))$ is a monotone increasing function of t in the intervals $[t_1, \bar{t}_1]$.

Proof: For $t \in [t_1, \bar{t}_1]$, we have

$$\begin{aligned} \hat{\theta}_5(t) &= \hat{\theta}_5(t_1) + \int_{t_1}^{t} \mu_{F_5}(x(\tau))x(\tau)d\tau \\ &= \hat{\theta}_5(t_1) + \int_{t_i}^{t} \left[2x(\tau) - 1 \right] x(\tau)d\tau \end{aligned} \quad (48)$$

and

$$\begin{aligned}\hat{\theta}_4(t) &= \hat{\theta}_4(t_1) + \int_{t_1}^{t} \mu_{F_4}(x(\tau))x(\tau)d\tau \\ &= \hat{\theta}_4(t_1) + \int_{t_1}^{t} [2-2x(\tau)]\, x(\tau)d\tau \quad (49)\end{aligned}$$

Therefore, the difference $\hat{\theta}_5(t)-\hat{\theta}_4(t)$ can be evaluated as

$$\hat{\theta}_5(t)-\hat{\theta}_4(t) = \hat{\theta}_5(t_1)-\hat{\theta}_4(t_1)+\int_{t_i}^{t} [4x(\tau)-3]\, x(\tau)d\tau \quad (50)$$

Since $x(t) \in [1-h, 1]$ for $t \in [t_1, \bar{t}_1]$ and $0 < h \leq \frac{1}{4} - \varepsilon_h$ as defined in (27), the integrand of the integral in (50) is greater than zero. Therefore, $\hat{\theta}_5(t)-\hat{\theta}_4(t)$ is a monotone increasing function for $t \in [t_1, \bar{t}_1]$. Actually, a lower bound of $\hat{\theta}_5(t) - \hat{\theta}_4(t)$ can be evaluated as

$$\hat{\theta}_5(t) - \hat{\theta}_4(t) \geq \hat{\theta}_5(t_1) - \hat{\theta}_4(t_1) + 4\varepsilon_h(1-h)(t-t_1) \quad (51)$$

Particularly, at $t = \bar{t}_1$, with (41), one can lead to

$$\begin{aligned}& \hat{\theta}_5(\bar{t}_1) - \hat{\theta}_4(\bar{t}_1) \\ \geq\ & \hat{\theta}_5(t_1) - \hat{\theta}_4(t_1) + 4\varepsilon_h(1-h)(\bar{t}_1 - t_1) \\ \geq\ & \hat{\theta}_5(t_1) - \hat{\theta}_4(t_1) + \frac{4\varepsilon_h(1-h)h}{f_{\max}}\end{aligned}$$

Moreover, from (51), it follows that if $\hat{\theta}_5(t_1) \geq \hat{\theta}_4(t_1)$, then $\hat{\theta}_5(t) \geq \hat{\theta}_4(t)$ for any $t \in [t_1, \bar{t}_1]$.

Now we show that $F(\xi(x(t)), \hat{\theta}(t))$ is a monotone increasing function of t in the intervals $[t_1, \bar{t}_1]$. Using the definitions of the membership functions in (**??**), we can get

$$\begin{aligned}& \hat{\theta}^T(t)\xi(x(t)) \\ =\ & [2x(t)-1]\left[\hat{\theta}_5(t) - \hat{\theta}_4(t)\right] + \hat{\theta}_4(t)\end{aligned}$$

Since both $\hat{\theta}_5(t)-\hat{\theta}_4(t)$ as well as $\hat{\theta}_4(t)$ are monotone increasing under the assumption $\hat{\theta}_5(t_1) \geq \hat{\theta}_4(t_1)$ and $x(t) \geq x(t_1) = 1-h$, it follows that inequality (47) holds. In addition, if $x(t)$ is also non-decreasing, then $F(\xi(x(t)), \hat{\theta}(t))$ is a monotone increasing function of t in the intervals $[t_1, \bar{t}_1]$. This completes the proof. ■

Lemma 8: Assume that a continuous switching of infinite times at the positive boundary $x = 1-h$ happens since a finite time $t = t_1$. Then there is an index I_1 such that $\hat{\theta}_5(t) \geq \hat{\theta}_4(t)$ for $t \geq t_{I_1}$.

Proof: Assume that there are two time sequences $\{t_i\}_{i=1}^{\infty}$ and $\{\bar{t}_i\}_{i=1}^{\infty}$ with $t_i < \bar{t}_i < t_{i+1}$ such that the adaptive fuzzy control law is used for $t \in [t_i, \bar{t}_i]$ with $x(t) \in [1-h, 1]$. It is noted that $x(t_i) = 1-h$ and $x(\bar{t}_i) = 1$ for any i. While keeping switching at the boundary $x = 1-h$, we shall repeatedly use the result in Lemma 7 and we shall show that there is a time instant t_{I_1} such that $\hat{\theta}_5(t_{I_1}) \geq \hat{\theta}_4(t_{I_1})$. In the following, we shall identify $\bar{t}_0$ as t_1. By using (46) for $t \in [t_i, \bar{t}_i]$, we have

$$\begin{aligned}& \hat{\theta}_5(\bar{t}_i) - \hat{\theta}_4(\bar{t}_i) \\ \geq\ & \hat{\theta}_5(t_i) - \hat{\theta}_4(t_i) + \frac{4\varepsilon_h(1-h)h}{f_{\max}} \\ =\ & \hat{\theta}_5(\bar{t}_{i-1}) - \hat{\theta}_4(\bar{t}_{i-1}) + \frac{4\varepsilon_h(1-h)h}{f_{\max}} \quad (52)\end{aligned}$$

for $i \geq 1$. By using the recursive inequality (52), one can get

$$\begin{aligned}& \hat{\theta}_5(t_{N+1}) - \hat{\theta}_4(t_{N+1}) \\ =\ & \hat{\theta}_5(\bar{t}_N) - \hat{\theta}_4(\bar{t}_N) \\ \geq\ & \hat{\theta}_5(t_1) - \hat{\theta}_4(t_1) + N4\varepsilon_h(1-h)\frac{h}{f_{\max}} \quad (53)\end{aligned}$$

Therefore, if we choose

$$N = \left\lceil \frac{\max\left(0, \hat{\theta}_4(t_1) - \hat{\theta}_5(t_1)\right) f_{\max}}{4\varepsilon_h(1-h)h} \right\rceil$$

where $\lceil x \rceil$ is the smallest integer with $x \leq \lceil x \rceil$, then, following from (53), $\hat{\theta}_5(t_{I_1}) \geq \hat{\theta}_4(t_{I_1})$ where we define $I_1 = N+1$. Consequentially, from (52), we $\hat{\theta}_5(t_i) \geq \hat{\theta}_4(t_i)$ for $i \geq I_1$. Therefore, by repeatedly using Lemma 7, we can conclude that $\hat{\theta}_5(t) \geq \hat{\theta}_4(t)$ for $t \geq t_{I_1}$. Note that if $\hat{\theta}_4(t_1) \leq \hat{\theta}_5(t_1)$, then $I_1 = 1$.

Lemma 9: It is impossible that a continuous switching of infinite times at the positive boundary $x = 1-h$ happens since a finite time $t = t_1$.

Proof: By contradiction, assume that a continuous switching of infinite times at the positive boundary $x = 1-h$ happens since a time instant $t = t_1$. By Lemma 8, there is an index I_1 such that $\hat{\theta}_5(t_i) \geq \hat{\theta}_4(t_i)$ for $i \geq I_1$. Then, by inequality (47) in Lemma 7, we have $F(x(t), \hat{\theta}(t)) = \hat{\theta}^T(t)\xi(x(t)) \geq F(x(t_i), \hat{\theta}(t_i))$ for $t \in [t_i, \bar{t}_i]$ with $i \geq I_1$. Therefore, for any positive integer N, we have

$$\begin{aligned}& x(\bar{t}_{N+I_1}) \\ =\ & x(t_{N+I_1}) + \int_{t_{N+I_1}}^{\bar{t}_{N+I_1}} \left[f(x(\tau)) - \hat{\theta}^T(\tau)\xi(x(\tau))\right] d\tau \\ \leq\ & 1-h \\ & + \int_{t_{N+I_1}}^{\bar{t}_{N+I_1}} \left[f_{\max} - \hat{\theta}^T(t_{N+I_1})\xi(x(t_{N+I_1}))\right] d\tau \quad (54)\end{aligned}$$

Now, following from inequality (42) in Lemma 6 and inequality (41), for any positive integer N, we have

$$\begin{aligned}& F(x(t_{N+I_1}), \hat{\theta}(t_{N+I_1})) \\ \geq\ & F(x(t_{I_1}), \hat{\theta}(t_{I_1})) + N\Delta_F \\ \geq\ & N\Delta_F\end{aligned}$$

and thus inequality (54) implies

$$\begin{aligned}& x(\bar{t}_{N+I_1}) \\ \leq\ & 1-h + \int_{t_{N+I_1}}^{\bar{t}_{N+I_1}} [f_{\max} - N\Delta_F]\, d\tau \quad (55) \\ =\ & 1-h + (f_{\max} - N\Delta_F)(\bar{t}_{N+I_1} - t_{N+I_1}) \quad (56)\end{aligned}$$

Suppose that we choose N such that

$$N > \left\lceil \frac{f_{\max}}{\Delta_F} \right\rceil$$

and thus $f_{\max} - N\Delta_F < 0$. Then, following from (41) and (56), we have

$$x(\bar{t}_{N+I_1}) \leq 1 - h + (f_{\max} - N\Delta_F)\,\frac{h}{f_{\max}} < 1 - h$$

However, the above inequality contradicts the assumption that a continuous switching of infinite times at the positive boundary $x = 1-h$ happens since $t = t_1$. This completes the proof. ■

Lemma 10: It is impossible that a switching at the positive boundary $x = 1 - h$ (or at the negative boundary $x = -(1-h)$) happens infinite times since any finite time $t = t_1$.

Proof: By contradiction, assume that a switch at the positive boundary $x = 1 - h$ (or at the negative boundary $x = -(1-h)$) happens infinite times at $t = t_i$ for $1 \leq i < \infty$ with $t_i < t_{i+1}$. Denote a time sequence $\bar{t}_i$ for $1 \leq i < \infty$ with $t_i < \bar{t}_i < t_{i+1}$ such that the adaptive fuzzy control law is applied in $[t_i, \bar{t}_i]$. Also define a time sequence $\check{t}_i$ for $1 \leq i < \infty$ such that the adaptive VSS control law is applied in $(\bar{t}_i, \check{t}_i)$ and $t_i < \bar{t}_i < \check{t}_i \leq t_{i+1}$ for $1 \leq i < \infty$. Then, since the adaptive fuzzy control is applied in $[t_i, \bar{t}_i]$, following from Lemma 6, we have

$$\begin{aligned}\hat{\theta}_5(\bar{t}_i) - \hat{\theta}_5(t_i) &= \int_{t_i}^{\bar{t}_i} \mu_{F_5}(x(\tau))x(\tau)d\tau \\ &\geq \mu_{F_5}(1-h)(1-h)\left(\bar{t}_i - t_i\right) \\ &\geq \mu_{F_5}(1-h)(1-h)\frac{h}{f_{\max}}\end{aligned}$$

Since $\hat{\theta}_5(t)$ is monotone increasing, i.e., $\hat{\theta}_5(t_{i+1}) \geq \hat{\theta}_5(\bar{t}_i)$ as $t_{i+1} \geq \bar{t}_i$, we have

$$\begin{aligned}\hat{\theta}_5(t_{i+1}) - \hat{\theta}_5(t_i) &\geq \hat{\theta}_5(\bar{t}_i) - \hat{\theta}_5(t_i) \\ &\geq \mu_{F_5}(1-h)(1-h)\frac{h}{f_{\max}}\end{aligned}$$

Therefore, recursively using the above inequality, one can obtain

$$\hat{\theta}_5(t_{N+1}) \geq \hat{\theta}_5(t_1) + N\mu_{F_5}(1-h)(1-h)\frac{h}{f_{\max}}$$

Since $\mu_{F_5}(x)$ is monotone increasing in the interval $[1-h, 1]$, for $t \in [t_{N+1}, \bar{t}_{N+1}]$, we have

$$\begin{aligned}&\hat{\theta}^T(t)\xi(x(t)) \\ =\; &\hat{\theta}_4(t)\mu_{F_4}(x(t)) + \hat{\theta}_5(t)\mu_{F_5}(x(t)) \\ \geq\; &\hat{\theta}_5(t)\mu_{F_5}(x(t)) \\ \geq\; &\hat{\theta}_5(t_{N+1})\mu_{F_5}(1-h) \\ \geq\; &\mu_{F_5}(1-h)\left[\hat{\theta}_5(t_1) + (1-h)\frac{h}{f_{\max}}N\mu_{F_5}(1-h)\right]\end{aligned}$$

Suppose that we choose N such that

$$N > \left\lceil \frac{\frac{f_{\max}}{\mu_{F_5}(1-h)} - \hat{\theta}_5(t_1)}{h(1-h)\mu_{F_5}(1-h)} f_{\max} \right\rceil$$

and thus

$$f_{\max} < \mu_{F_5}(1-h)\left[\hat{\theta}_5(t_1) + N\mu_{F_5}(1-h)(1-h)\frac{h}{f_{\max}}\right]$$

Then one can lead to

$$\begin{aligned}&x(\bar{t}_{N+1}) \\ =\; &x(t_{N+1}) + \int_{t_{N+1}}^{\bar{t}_{N+1}} \left[f(x(\tau)) - \hat{\theta}^T(\tau)\xi(x(\tau))\right] d\tau \\ <\; &1-h\end{aligned}$$

However, the above inequality contradicts the assumption that a switching at the positive boundary $x = 1-h$ happens infinite times since $t = t_1$. This completes the proof. ■

Lemma 11: Under the specified switching mechanism in (8)-(10) including the adaptive VSS control in (30) and the adaptive fuzzy control in (31), we have the following results.

(i) there is a finite time t_{f_0} such that $x(t) \in \Omega_x$ and the adaptive fuzzy control is used for $t \geq t_{f_0}$ and

(ii) the parameters $\hat{c}_1(t)$ and $\hat{c}_2(t)$ in the adaptive VSS control are bounded for $t \in [0, \infty)$.

Proof: (Part i) Suppose that $x(0) > 1-h$. Then the adaptive VSS control in (30) will ensure that there is a finite time t_1 such that $x(t_1) = 1-h$ and $x(t) > 1-h$ for $t \in [0, t_1)$ as stated in Lemma 2. At $t = t_1$, the adaptive fuzzy control law in (31) will then be applied. When applying the adaptive fuzzy control law at some time instant $t = t_s$, the case that a continuous switching of infinite times at the positive boundary $x = 1-h$ since $t = t_s$ has been excluded by Lemma 9. Therefore, there are three possible cases at $t = t_s$.

(A1) There is no control law switching happened at the boundary $x = 1-h$ since $t = t_s$, i.e., $\{x(t)\,|t \geq t_s\} \subset \Omega_x = [-1, 1]$.

(A2) There is no control law switching happened at the boundary $x = 1-h$ at $t = t_s$ and there are switching operations after $t = t_s$.

(A3) A continuous switching of finite times happens at the positive boundary $x = 1-h$ since $t = t_s$.

The above three cases are also applied to the situation that $x(0) < -(1-h)$. If $|x(0)| \leq 1-h$, then the adaptive fuzzy control will be applied at $t = 0$ and there are two possible cases.

(B1) There is no switching of control laws for $t \geq 0$, i.e., $\{x(t)\,|t \geq 0\} \subset \Omega_x = [-1, 1]$.

(B2) There are switching operations after $t = 0$.

Combining the situations (A1)-(A3) and (B1)-(B2), we can conclude that there are two possibilities for the trajectory $\{x(t), t \geq 0\}$.

(C1) There is a finite time t_{f_0} such that $x(t) \in \Omega_x$ and the adaptive fuzzy control is used for $t \geq t_{f_0}$.

(C2) A switching of control law at either $x = 1-h$ or $x = -(1-h)$ happens infinite times.

Note that Cases (C1) and (C2) are mutually exclusive, since the time to complete a switching is greater than or equal to a constant $h/f_{\max}$ as indicated in Lemma 6. However, Case (C2) is excluded by Lemma 10 and thus the result in part (i) is confirmed.

(Part ii) Note that a switching at either $x = 1 - h$ or $x = -(1-h)$ can only happen for finite times following from part (i). Define a set of time intervals $\left\{\left(\bar{t}_i, \check{t}_i\right)\right\}_{i=1}^{N_V}$ where N_V is a finite positive integer, $\sum_{i=1}^{N_V}\left(\check{t}_i - \bar{t}_i\right) < \infty$, and the adaptive VSS control is only applied in $\left(\bar{t}_i, \check{t}_i\right)$ for $1 \le i \le N_V$. Then, following from (12) and (13), we have

$$\begin{aligned}
\hat{c}_1(\check{t}_i) - \hat{c}_1(\bar{t}_i) &= \int_{\bar{t}_i}^{\check{t}_i} \Gamma_1 \left|x(t)\right|^2 dt \\
&\le \Gamma_1 x_{i,\max}^2 (\check{t}_i - \bar{t}_i) \\
\hat{c}_2(\check{t}_i) - \hat{c}_2(\bar{t}_i) &= \int_{\bar{t}_i}^{\check{t}_i} \Gamma_1 \left|x(t)\right|^3 dt \\
&\le \Gamma_1 x_{i,\max}^3 (\check{t}_i - \bar{t}_i)
\end{aligned}$$

where

$$x_{i,\max} = \sup_{t \in \left(\bar{t}_i, \check{t}_i\right)} |x(t)|$$

Since $\hat{c}_1(t)$ and $\hat{c}_2(t)$ are kept invariant when the adaptive fuzzy control law is applied, we have

$$\begin{aligned}
\hat{c}_1(\bar{t}_i) &= \hat{c}_1(\check{t}_{i-1}) \\
\hat{c}_2(\bar{t}_i) &= \hat{c}_2(\check{t}_{i-1})
\end{aligned}$$

and thus

$$\begin{aligned}
& \hat{c}_1(\check{t}_{N_V}) \\
\le\ & \hat{c}_1(\bar{t}_1) + \Gamma_1 \left(\max_{1\le i \le N_V} x_{i,\max}^2\right) \sum_{i=1}^{N_V} (\check{t}_i - \bar{t}_i) \\
<\ & \infty
\end{aligned}$$

amd

$$\begin{aligned}
& \hat{c}_2(\check{t}_{N_V}) \\
\le\ & \hat{c}_2(\bar{t}_1) + \Gamma_1 \left(\max_{1\le i \le N_V} x_{i,\max}^3\right) \sum_{i=1}^{N_V} (\check{t}_i - \bar{t}_i) \\
<\ & \infty
\end{aligned}$$

Since the adaptive fuzzy control is applied for $t \ge \check{t}_{N_V}$, we have $\hat{c}_1(t) = \hat{c}_1(\check{t}_{N_V})$ and $\hat{c}_2(t) = \hat{c}_2(\check{t}_{N_V})$ for $t \ge \check{t}_{N_V}$. This completes the proof. ■

B. Convergence analysis

Following from Lemma 11, the adaptive fuzzy control is applied for $t \ge t_{f_0}$ where t_{f_0} is a finite time and $x(t) \in \Omega_x$ for $t \ge t_{f_0}$. Recall that $\{\Omega_{x,i}\}_{i=1}^{2L-2}$ is a partition of the universe of discourse Ω_x. Particularly, let $\Omega_{x,0} = \Omega_{x,L-1} \cup \Omega_{x,L} = \left[\gamma_{\frac{L-1}{2}}, \gamma_{\frac{L+1}{2}}\right]$ and thus $0 \in \Omega_{x,0}$. Now define sets S_i for $1 \le i \le 2L-2$ such that

$$S_i = \{t \,| x(t) \in \Omega_{x,i},\ t \ge t_{f_0}\}$$

and denote the time length $\sigma(S_i)$ as the Borel measure of the set S_i.

Lemma 12: The the time length $\sigma(S_i)$ is finite for $1 \le i \le L-3$ and $L+2 \le i \le 2L-2$.Therefore there is a finite time t_f with $t_f \ge t_{f_0}$ such that $x(t) \in \Omega_{x,0} = \cup_{i=L-2}^{L+1} \Omega_{x,i}$ and the adaptive control is used for $t \ge t_f$.

Proof: (Part i: for $i = 1$ and $i = 2L-2$) First, by contradiction, assume that $\sigma(S_{2L-2}) = \infty$. In this case, there are two possibilities: (i) there is a finite time t_a such that $x(t) \in \Omega_{x,2L-2}$ for $t \ge t_a \ge t_{f_0}$ and (ii) the trajectory $x(t)$ keeps visiting the connected region $\Omega_{x,2L-2}$ in infinite disjoined time intervals. We can use the same methodology as done in the proof of Lemma 5 to verify that the first possibility is impossible. In the following, we shall also show that the second possibility is impossible and thus $\sigma(S_{2L-2}) < \infty$. For the second possibility, since $x(t)$ is a continuous function of t, there is a subset $\cup_{i=1}^{\infty}(t_i, \bar{t}_i) \subset S_{2L-2}$ such that $t_{f_0} \le t_i < \bar{t}_i \le t_{i+1}$, $x(t_i) = \gamma_{L-1}$, and $\sum_{i=1}^{\infty}(\bar{t}_i - t_i) = \infty$. Note that for any $x(t) \in \Omega_{x,2L-2}$, only the $(L-1)-$th and the $L-$th rules of the fuzzy system will be fired and thus

$$\hat{\theta}^T(t)\xi(x(t)) = \hat{\theta}_{L-1}(t)\mu_{F_{L-1}}(x(t)) + \hat{\theta}_L(t)\mu_{F_L}(x(t))$$

Also as $\Omega_{x,2L-2} \subset [0,1]$, following from Lemma 3, we have $\hat{\theta}_{L-1}(t) \ge 0$, $\hat{\theta}_L(t) \ge 0$, and both $\hat{\theta}_{L-1}(t)$ as well as $\hat{\theta}_L(t)$ are monotone increasing functions of t. Moreover, for $x(t) \in \Omega_{x,2L-2}$, i.e., $\gamma_{L-1} < x(t) \le d_L$, we have $\mu_{F_{L-1}}(x(t))$ is monotone decreasing with $0 \le \mu_{F_{L-1}}(x(t)) < \frac{1}{2}$ and $\mu_{F_L}(x(t))$ is monotone increasing with $\frac{1}{2} < \mu_{F_L}(x(t)) \le 1$. Now for $[t_i, \bar{t}_i]$,, we have

$$\begin{aligned}
\hat{\theta}_L(\bar{t}_i) - \hat{\theta}_L(t_i) &= \int_{t_i}^{\bar{t}_i} \mu_{F_L}(x(\tau))x(\tau)d\tau \\
&\ge \mu_{F_L}(\gamma_{L-1})\gamma_{L-1}\left(\bar{t}_i - t_i\right)
\end{aligned}$$

Since $\hat{\theta}_L(t)$ is monotone increasing, i.e., $\hat{\theta}_L(t_{i+1}) \ge \hat{\theta}_L(\bar{t}_i)$ as $t_{i+1} \ge \bar{t}_i$, we have

$$\begin{aligned}
\hat{\theta}_L(t_{i+1}) - \hat{\theta}_L(t_i) &\ge \hat{\theta}_L(\bar{t}_i) - \hat{\theta}_L(t_i) \\
&\ge \mu_{F_L}(\gamma_{L-1})\gamma_{L-1}\left(\bar{t}_i - t_i\right)
\end{aligned}$$

Therefore, recursively using the above inequality, one can obtain

$$\hat{\theta}_L(t_{N+1}) \ge \hat{\theta}_L(t_1) + \mu_{F_L}(\gamma_{L-1})\gamma_{L-1}\sum_{i=1}^{N}\left(\bar{t}_i - t_i\right)$$

Since $\mu_{F_L}(x)$ is monotone increasing in the interval $\left[\gamma_{L-1}, d_L\right]$, for $t \in [t_{N+1}, \bar{t}_{N+1}]$, we have

$$\begin{aligned}
& \hat{\theta}^T(t)\xi(x(t)) \\
=\ & \hat{\theta}_{L-1}(t)\mu_{F_{L-1}}(x(t)) + \hat{\theta}_L(t)\mu_{F_L}(x(t)) \\
\ge\ & \hat{\theta}_L(t)\mu_{F_L}(x(t)) \\
\ge\ & \hat{\theta}_L(t_{N+1})\mu_{F_L}(\gamma_{L-1}) \\
\ge\ & \mu_{F_L}(\gamma_{L-1})\Big[\hat{\theta}_L(t_1) \\
& + \mu_{F_L}(\gamma_{L-1})\gamma_{L-1}\sum_{i=1}^{N}\left(\bar{t}_i - t_i\right)\Big]
\end{aligned}$$

Suppose that we choose N such that

$$\sum_{i=1}^{N}(\bar{t}_i - t_i) > \frac{\frac{f_{\max}}{\mu_{F_L}(\gamma_{L-1})} - \hat{\theta}_L(t_1)}{\mu_{F_L}(\gamma_{L-1})\gamma_{L-1}}$$

Then one can lead to

$$\begin{aligned} & x(\bar{t}_{N+1}) \\ = \ & x(t_{N+1}) + \int_{t_{N+1}}^{\bar{t}_{N+1}} \left[f(x(\tau)) - \hat{\theta}^T(\tau)\xi(x(\tau))\right] d\tau \\ \leq \ & \gamma_{L-1} + \int_{t_{N+I_1}}^{\bar{t}_{N+I_1}} \left\{f_{\max} - \mu_{F_L}(\gamma_{L-1})\left[\hat{\theta}_L(t_1)\right.\right. \\ & \left.\left. + \mu_{F_L}(\gamma_{L-1})\gamma_{L-1}\sum_{i=1}^{N}(\bar{t}_i - t_i)\right]\right\} d\tau \\ < \ & \gamma_{L-1} \end{aligned}$$

which contradicts the assumption that $x(\bar{t}_{N+1}) \geq \gamma_{L-1}$. Therefore, we have $\sigma(S_{2L-2}) < \infty$. The above analysis can be also applied to the case $i = 1$ to show that $\sigma(S_1) < \infty$.

(Part ii: for $i = 2$ and $i = 2L - 3$)

First, by contradiction, assume that $\sigma(S_{2L-3}) = \infty$. In this case, there are two possibilities: (i) there is a finite time t_a such that $x(t) \in \Omega_{x,2L-3}$ for $t \geq t_a \geq t_{f_0}$ and (ii) the trajectory $x(t)$ keeps visiting the connected region $\Omega_{x,2L-3}$ in infinite disjoined time intervals. We can use the same methodology as done in the proof of Lemma 5 to verify that the first possibility is impossible. In the following, we shall also show that the second possibility is impossible and thus $\sigma(S_{2L-3}) < \infty$. For the second possibility, since $x(t)$ is a continuous function of t, there is a subset $\cup_{i=1}^{\infty}(t_i, \bar{t}_i) \subset S_{2L-3}$ such that $t_f \leq t_i < \bar{t}_i \leq t_{i+1}$, $x(t_i) = d_{L-1}$, and $\sum_{i=1}^{\infty}(\bar{t}_i - t_i) = \infty$. Note that for any $x(t) \in \Omega_{x,2L-3}$, only the $(L-1)-$th and the $L-$th rules of the fuzzy system will be fired and thus

$$\hat{\theta}^T(t)\xi(x(t)) = \hat{\theta}_{L-1}(t)\mu_{F_{L-1}}(x(t)) + \hat{\theta}_L(t)\mu_{F_L}(x(t))$$

Also as $\Omega_{x,2L-3} \subset [0, 1]$, following from Lemma 3, we have $\hat{\theta}_{L-1}(t) \geq 0$, $\hat{\theta}_L(t) \geq 0$, and both $\hat{\theta}_{L-1}(t)$ as well as $\hat{\theta}_L(t)$ are monotone increasing function of t. Moreover, for $x(t) \in \Omega_{x,2L-3}$, i.e., $d_{L-1} < x(t) \leq \gamma_{L-1}$, we have $\mu_{F_{L-1}}(x(t))$ is monotone decreasing with $\frac{1}{2} \leq \mu_{F_{L-1}}(x(t)) < 1$ and $\mu_{F_L}(x(t))$ is monotone increasing with $0 < \mu_{F_L}(x(t)) \leq \frac{1}{2}$. Now for $[t_i, \bar{t}_i]$,, we have

$$\begin{aligned} \hat{\theta}_{L-1}(\bar{t}_i) - \hat{\theta}_{L-1}(t_i) &= \int_{t_i}^{\bar{t}_i} \mu_{F_{L-1}}(x(\tau))x(\tau)d\tau \\ &\geq \mu_{F_{L-1}}(\gamma_{L-1})d_{L-1}(\bar{t}_i - t_i) \end{aligned}$$

Since $\hat{\theta}_{L-1}(t)$ is monotone increasing, i.e., $\hat{\theta}_{L-1}(t_{i+1}) \geq \hat{\theta}_{L-1}(\bar{t}_i)$ as $t_{i+1} \geq \bar{t}_i$, we have

$$\begin{aligned} \hat{\theta}_{L-1}(t_{i+1}) - \hat{\theta}_{L-1}(t_i) &\geq \hat{\theta}_{L-1}(\bar{t}_i) - \hat{\theta}_{L-1}(t_i) \\ &\geq \mu_{F_{L-1}}(\gamma_{L-1})d_{L-1}(\bar{t}_i - t_i) \end{aligned}$$

Therefore, recursively using the above inequality, one can obtain

$$\begin{aligned} & \hat{\theta}_{L-1}(t_{N+1}) \\ \geq \ & \hat{\theta}_{L-1}(t_1) + \mu_{F_{L-1}}(\gamma_{L-1})d_{L-1}\sum_{i=1}^{N}(\bar{t}_i - t_i) \end{aligned}$$

Since $\mu_{F_{L-1}}(x)$ is monotone decreasing in the interval $[d_{L-1}, \gamma_{L-1}]$, for $t \in [t_{N+1}, \bar{t}_{N+1}]$, we have

$$\begin{aligned} & \hat{\theta}^T(t)\xi(x(t)) \\ = \ & \hat{\theta}_{L-1}(t)\mu_{F_{L-1}}(x(t)) + \hat{\theta}_L(t)\mu_{F_L}(x(t)) \\ \geq \ & \hat{\theta}_{L-1}(t)\mu_{F_{L-1}}(x(t)) \\ \geq \ & \hat{\theta}_{L-1}(t_{N+1})\mu_{F_{L-1}}(\gamma_{L-1}) \\ \geq \ & \mu_{F_L}(\gamma_{L-1})\left[\hat{\theta}_{L-1}(t_1)\right. \\ & \left. + \mu_{F_{L-1}}(\gamma_{L-1})d_{L-1}\sum_{i=1}^{N}(\bar{t}_i - t_i)\right] \end{aligned}$$

Suppose that we choose N such that

$$\sum_{i=1}^{N}(\bar{t}_i - t_i) > \frac{\frac{f_{\max}}{\mu_{F_{L-1}}(\gamma_{L-1})} - \hat{\theta}_{L-1}(t_1)}{\mu_{F_{L-1}}(\gamma_{L-1})d_{L-1}}$$

Then one can lead to

$$\begin{aligned} & x(\bar{t}_{N+1}) \\ = \ & x(t_{N+1}) + \int_{t_{N+1}}^{\bar{t}_{N+1}} \left[f(x(\tau)) - \hat{\theta}^T(\tau)\xi(x(\tau))\right] d\tau \\ \leq \ & d_{L-1} + \int_{t_{N+I_1}}^{\bar{t}_{N+I_1}} \left\{f_{\max} - \mu_{F_{L-1}}(\gamma_{L-1})\left[\hat{\theta}_{L-1}(t_1)\right.\right. \\ & \left.\left. + \mu_{F_{L-1}}(\gamma_{L-1})d_{L-1}\sum_{i=1}^{N}(\bar{t}_i - t_i)\right]\right\} d\tau \\ < \ & d_{L-1} \end{aligned}$$

which contradicts the assumption that $x(\bar{t}_{N+1}) \geq d_{L-1}$. Therefore, we have $\sigma(S_{2L-3}) < \infty$. The above analysis can be also applied to the case $i = 2$ to show that $\sigma(S_2) < \infty$.

(Part iii, the other cases) The other cases can be treated by the same methods as done in the previous two parts except for the cases $i = L - 1$ and $i = L$. This completes the proof. ∎

Theorem 5: Assume that $f(x)$ is a convex function in $[0, 1]$ and a concave function in $[-1, 0]$. Then $\lim_{t\to\infty} x(t) = 0$ and $\hat{\theta}(t)$ is bounded over $[0, \infty)$.

Proof: Note that if the fuzzy system $F(x, \theta)$ consists of a fuzzy singleton, Mamdani product inference engine, center average defuzzifier, and triangular member ship functions, then $\theta_i^* = f(d_i)$ for $1 \leq i \leq L$, $F(d_i, \theta^*) = f(d_i)$ for $1 \leq i \leq L$, $F(x, \theta)$ is a piecewise linear approximator of $f(x)$ in Ω_x [10]. If $f(x)$ is a convex function in $[0, 1]$, then

$$f(x) - \xi^T(x)\theta^* \leq 0, \text{ for } x \in [0, 1] \quad (57)$$

On the other hand, if $f(x)$ is a concave function in $[-1, 0]$, then

$$f(x) - \xi^T(x)\theta^* \geq 0, \text{ for } x \in [-1, 0] \quad (58)$$

Now consider the following positive definite function

$$V_a = \frac{1}{2}x^2 + \frac{1}{2}\tilde{\theta}^T\tilde{\theta}$$

By applying the closed-loop dynamic equation (31), we get

$$\begin{aligned} \dot{V}_a &= x\left[f(x) - \hat{\theta}^T\xi(x)\right] + \tilde{\theta}^T\dot{\tilde{\theta}} \\ &= x\left[f(x) - \xi^T(x)\theta^* - \xi^T(x)\tilde{\theta}\right] + \tilde{\theta}^T[\xi x] \\ &= x\left[f(x) - \xi^T(x)\theta^*\right] \leq 0 \quad (59) \end{aligned}$$

where we have invoked inequalities (57) and (58). Inequality (59) implies that $\hat{\theta}(t)$ is bounded over $[t_f, \infty)$. Note that $\hat{\theta}(t)$ is also bounded over $[0, t_f)$.

Now we consider another positive definite function

$$V_2 = \frac{1}{2}x^2 + \frac{1}{2}\hat{\theta}^T\hat{\theta} \quad (60)$$

The time derivative of V_2 along the system trajectory of the closed-loop system dynamics defined in (31) is given by

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2}x\dot{x} + \frac{1}{2}\hat{\theta}^T\dot{\hat{\theta}} \\ &= x\left[f(x) - \hat{\theta}^T\xi(x)\right] + \hat{\theta}^T\xi x \\ &= xf(x) \geq 0 \end{aligned}$$

By the symmetric property of $f(x)$ in (5), we have

$$\begin{cases} \dot{V}_2 > 0, & \text{if } x \neq 0 \\ \dot{V}_2 = 0, & \text{if } x = 0 \end{cases}$$

which implies that $V_2(t)$ is non-decreasing for $t \geq t_f$. Since $\hat{\theta}(t)$ and $x(t)$ are bounded over $[t_f, \infty)$, $V_2(t)$ converges to a finite positive limit . If $V_2(t)$ converges to a finite positive limit, i.e.,

$$\lim_{t\to\infty} V_2(t) = C_{V_2}$$

then, by the monotone increasing property of $V_2(t)$, we have

$$\frac{1}{2}x^2(t) + \frac{1}{2}\hat{\theta}(t)^T\hat{\theta}(t) \leq C_{V_2}$$

and thus

$$\begin{aligned} |x(t)| &\leq \sqrt{2C_{V_2}} \\ \left\|\hat{\theta}(t)\right\| &\leq \sqrt{2C_{V_2}} \end{aligned}$$

Now compute the second-order differential of $V_2(t)$ as the follows

$$\begin{aligned} \ddot{V}_2(t) &= \dot{x}f(x) + x\frac{df(x)}{dx}\dot{x} \\ &= \left(f(x) + x\frac{df(x)}{dx}\right)\left(f(x) - \xi^T(x)\hat{\theta}\right) \end{aligned}$$

It is easy to see that $\ddot{V}_2(t)$ is a bounded function of t from the following inequality

$$\begin{aligned} &\left|\ddot{V}_2(t)\right| \\ \leq\ &\left|f(x) + x\frac{df(x)}{dx}\right|\left|f(x) - \xi^T(x)\hat{\theta}\right| \\ \leq\ &\left(|f(x)| + |x|\left|\frac{df(x)}{dx}\right|\right)\left(|f(x)| + \left|\xi^T(x)\hat{\theta}\right|\right) \\ \leq\ &(f_{\max} + \kappa_f)\left(f_{\max} + \left\|\hat{\theta}(t)\right\|\|\xi(x)\|\right) \\ \leq\ &(f_{\max} + \kappa_f)\left(f_{\max} + \sqrt{2C_{V_2}}\right) \end{aligned}$$

and thus $\dot{V}_2(t)$ is uniformly continuous. Then, by Barbalat's lemma [9], we get

$$\lim_{t\to\infty} \dot{V}_2(t) = 0$$

which is equivalent to

$$\lim_{t\to\infty} x(t)f(x(t)) = 0$$

Since, by (5), $xf(x) = 0$ if and only if $x = 0$, the above equation implies that

$$\lim_{t\to\infty} x(t) = 0$$

This completes the proof. ■

Example 1: In the example, we consider the case

$$f(x) = 2x + 2x|x| \quad (61)$$

and thus

$$\begin{bmatrix} c_1^* & c_2^* \end{bmatrix} = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

We note that the plant in this example is a highly unstable system. Therefore, with the initial state $x(0) = 5$, the state $x(t)$ usually bursts within very short time interval without a proper feedback control. The settings of the adaptive VSS controller are given as

$$\begin{aligned} r &= 1 \\ \hat{c}_1(0) &= 0 \\ \hat{c}_2(0) &= 0 \end{aligned}$$

and those of the adaptive fuzzy controller are given as

$$\begin{aligned} \hat{\theta}(0) &= 0 \\ \Gamma_2 &= 10I \end{aligned}$$

Note that in the derivation of the main result, the learning rate matrix Γ_2 is set as I. Actually, the main results hold for any positive definite matrix Γ_2. The hysteresis size h is given as $\frac{1}{5}$ such that $1 - h = \frac{4}{5}$. The responses of $x(t)$, $\hat{c}_i(t)$, $u(t)$ are shown in Fig. 3, Fig. 4, and Fig. 5, respectively. For a very short time period after $t = 0$ before $\hat{c}_1(t)$ and $\hat{c}_2(t)$ are large enough, the closed-loop system is unstable and the state $x(t)$ bursts faster and faster. However, such a high amplitude of $x(t)$ will also rapidly increase the values of $\hat{c}_1(t)$ and $\hat{c}_2(t)$ as shown in Fig. 4. This also results in a high peak of $u(t)$ as shown in Fig. 5, which in turns tends to stabilizes the system. When $\hat{c}_1(t)$ and

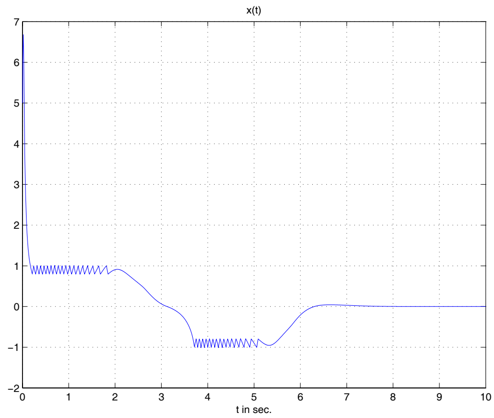

Fig. 3. The evolution of the state $x(t)$ in Example 1 with $x(0) = 5$.

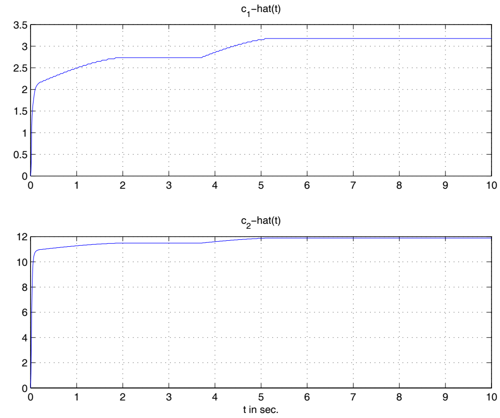

Fig. 4. The responses of $\hat{c}_1$and $\hat{c}_2$ in Example 1 with $\hat{c}_1(0) = 0$ and $\hat{c}_2(0) = 0$.

$\hat{c}_2(t)$ are large enough such that $\dot{x}(t_p) < 0$ at some time t_p, with $t_p < 0.1$ seconds, then $x(t)$ and thus $u(t)$ begin to decrease and the growth of $\hat{c}_1(t)$ and $\hat{c}_2(t)$ will slow down. At some time t_1 with $t_1 = 0.21$ seconds, $x(t)$ hits the switching boundary $x = 1 - h$.

After $t = t_1$, the adaptive fuzzy controller will take over the system. However, since the parameter vector $\hat{\theta}$ is not well trained yet, the system remains unstable and the state $x(t)$ still tries to escape away from the switching boundary. However, when $x(\bar{t}_1) = 1$ at some time $\bar{t}_1$, the adaptive VSS controller will be in charge of the system and $x(t)$ will be forced back to $x(t_2) = 1$ at some time t_2. Until the parameter vector $\hat{\theta}$, actually $\hat{\theta}_4$ and mainly $\hat{\theta}_5$, is well trained as shown in Fig. 6, there is a continuous switching of 18 times occurred at the boundary $x = 1 - h$, as observed in Fig. 3. Within the time period of this continuous switching, the value of the fuzzy system $F(x, \hat{\theta})$ evaluated at $t = t_i$ when a switching occurs increases as i increases. When $f(x(t_i)) - F(x(t_i), \hat{\theta}(t_i))$ is too large, then a switching $t = t_i$ will be inevitable. At the end of this continuous switching, the adaptive fuzzy control is applied again at $t = t_{19} = 1.85$. At some time t_q after $t = t_{19}$, we have $f(x(t_q)) < F(x(t_q), \hat{\theta}(t_q))$ and $x(t)$ begins to decay to zero. When $x(t) = 0$, $F(x(t), \hat{\theta}(t))$ is still larger than zero. Since $f(x) < 0$ for $x < 0$ in this example, and $\hat{\theta}_1$ as well as $\hat{\theta}_2$ have not been well trained until now, $x(t)$ will be pass through $x = 0$ and reach another boundary $x = -1$. Then, adaptive VSS control takes over the system again and another continuous switching of finite time at $x = -(1 - h)$ happens. After the second continuous switching, there is no switching any more and $x(t)$ converges to zero asymptotically. From Fig. 5, we see that the amplitude of the control signal $u(t)$ of the adaptive fuzzy controller is much smaller than that of the adaptive VSS controller. That is why we say that the adaptive VSS control is for coarse control and the adaptive fuzzy control is suitable for fine control.

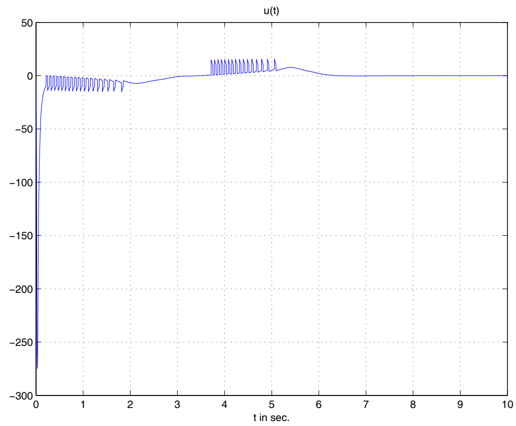

Fig. 5. The evolution of the control input $u(t)$ in Example 1.

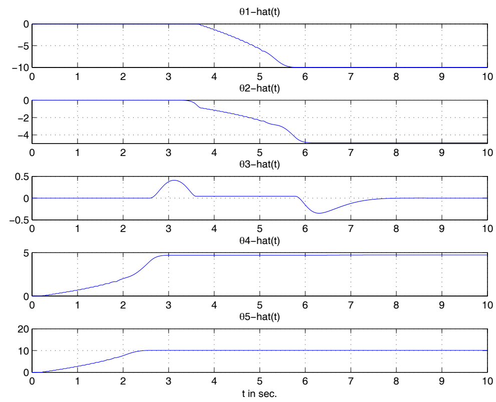

Fig. 6. The responses of $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, $\hat{\theta}_3(t)$, $\hat{\theta}_4(t)$ and $\hat{\theta}_5(t)$ in Example 1 with $\hat{\theta}(0) = 0$.

VI. Conclusion and Discussion

In the field of adaptive fuzzy control, there has been a severe deficiency by assuming the premise variables will usually stay within the universe of discourse in the derivation of stability of the adaptive control system. To overcome this deficiency, we develop a switching adaptive control scheme using only essential qualitative information of the plant to attain asymptotical stability of the adaptive control system for a typical first-order nonlinear system without imposing the mentioned severe assumption. The switching adaptive control system consists of an adaptive VSS controller for coarse control, an adaptive fuzzy controller for fine control, and a hysteresis switching mechanism. An adaptive VSS control scheme is proposed to force the state to enter the universe of discourse in finite time. While the premise variable is within the universe of discourse, an adaptive fuzzy control is proposed to learn the capability to stabilize the plant. At the boundary of the universe of discourse, a hysteresis switching scheme between the two controllers will be proposed. We show that after finite times of switching, the premise variables of the fuzzy system will remain within the universe of discourse and stability of the closed-loop system can be attained by applying Lyapunov direct method.

Future studies of this work are described as follows.

(i) Extension of the proposed scheme to nonlinear systems with strict feedback form

(ii) Extension to framework of stochastic control systems.

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行政院國家科學委員會補助國內專家學者出席國際學術會議報告

99 年 07 月 23 日

附件三

報告人姓名	李柏坤	服務機構及職稱	中華大學電機系　教授
時間 會議 地點	98 年 7 月 11 日至 14 日 中國山東青島	本會核定補助文號	國科會 NSC 98-2221-E-216 -032
會議名稱	（中文）2010 年機器學習與人工頭腦學國際研討會 (ICMLC 2010) （英文）2010 International Conference on Machine Learning and Cybernetics		
發表論文題目	（中文）隨機調節$H\infty$控制應用於 HIV 治療之研究 （英文）The Applications of Stochastic Regulation $H\infty$ Control to HIV Therapy		

表 Y04

一、參加會議經過:

此次2010年機器學習與人工頭腦學國際研討會(2010 International Conference on Machine Learning and Cybernetics ，ICMLC 2010)，由河北大學、華南理工大學、IEEE SMC (System, Man, and Cybernetics) 協會等單位聯合主辦，於 98 年 7 月 11 日到 98 年 7 月 14 日，在中國山東青島市之海信洲際酒店舉行。台灣的學者參與此研討會非常踴躍。

此次研討會所有論文都列入 IEEE Explorer 之資料庫，都屬於 EI Index。研討會之網路首頁為 http://www.icmlc.com/，整個研討會包含二個 Plenary Talk：

[1] Multiple Kernel Learning and Feature Space Denoising, Speaker: Professor Josef Kittler, Director of the Centre for Vision, Speech and Signal Processing, Faculty of Engineering and Physical Sciences, University of Surrey, UK.

[2] Incompleteness in Data for Decision Making, Speaker: Professor Bryan Scotney, Professor of Informatics and Director of the Computer Science Research Institute, University of Ulster, UK.

以及一個 Panel Discussion，題目為 Teach an Old Dog to Do New Tricks -- Learning and Recognizing a World of Problems。另外有兩個 Tutorials：

[1] Multiple Classifier Systems, Speaker: Prof. Fabio Roli.

[2] How to disseminate your research results: essentials of effective publishing, Speaker: Prof. Witold Pedrycz.

此次研討會之主題包含：

1. Adaptive systems
2. Business intelligence
3. Biometrics
4. Bioinformatics
5. Data and web mining
6. Intelligent agent
7. Financial engineering
8. Inductive learning
9. Geoinformatics
10. Pattern Recognition
11. Logistics
12. Intelligent control
13. Media computing
14. Neural net and support vector machine
15. Hybrid and nonlinear system
16. Fuzzy set theory, fuzzy control and system
17. Knowledge management
18. Information retrieval
19. Intelligent and knowledge based system
20. Rough and fuzzy rough set

21. Networking and information security
22. Evolutionary computation
23. Ensemble method
24. Information fusion
25. Visual information processing
26. Computational life science

二、與會心得

(1) 從此次研討會所安排之主題來看，比較偏向人工智能於資訊工程之研究，各國有關人工智慧理論都有顯著的研究成果，幾個比較新的主題如 Media computing、Bioinformatics、Computational life science、Business intelligence，非常值得國內學界注意其發展。

(2) 除了認識許多中國之學者外，也認識了很多來自全世界各地的菁英學者，對於將來推動國際學術交流，有相當大的幫助。

(3) 大陸在人工智能領域之研究成果亦有長足之進步，在 IEEE SMC Society 之影響力也已超過台灣相關學界，國內應該即起直追。

三、考察參觀活動(無是項活動者省略)

7 月 13 日上午，應青島大學自動化學院邀請，由元智大學電機工程系系主任陳永盛教授帶隊的臺灣學術代表團到青島大學參觀訪問。

應青島大學自動化學院負責人向臺灣代表團介紹了學院的基本情況，其中自動化學院之發展重點為控制理論與電力電子；臺灣代表團各位教授也分別介紹了各自學校及所在學院的基本情況，。隨後代表團首先參觀了青島大學自動化學院的國家級電工自動化學院，此實驗室包含基本電工實驗室、電力實驗室、電子實驗室、PLC 控制實驗室，學生於畢業前須通過相當多之實驗課程，為學生實作能力打下深厚之基礎，臺灣學術代表團並與實驗室師生進行了學術交流，就共同感興趣的學術問題進行了有益的探討。

四、建議

台灣應該多爭取舉辦國際研討會，使得全世界各地的菁英學者，能夠共聚一堂。大陸學術界的國際化，已經逐步生根，同時以此為基礎邁向國際之競爭，台灣學術界的國際化還有待大家的努力。

五、攜回資料名稱及內容

(1) 完整論文光碟片。

六、其他

無。

無衍生研發成果推廣資料

98 年度專題研究計畫研究成果彙整表

計畫主持人：李柏坤			計畫編號：98-2221-E-216-032-				
計畫名稱：基於 T-S 模糊模型之隨機非線性韌性適應控制							
成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	1	1	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	1	1	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	2	2	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	1	1	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	2	2	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	2	2	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	以專利’’具有無線識別標籤及偵測包裝開封之機制’’，獲得 2010 年日內瓦發明展金牌獎。

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

   ■達成目標

   □未達成目標（請說明，以 100 字為限）

   　　□實驗失敗

   　　□因故實驗中斷

   　　□其他原因

   說明：

2. 研究成果在學術期刊發表或申請專利等情形：

   論文：□已發表 □未發表之文稿 ■撰寫中 □無

   專利：□已獲得 □申請中 ■無

   技轉：□已技轉 □洽談中 ■無

   其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

   於未來幾年我們將探討命定性及隨機模糊系統之韌性適應控制，我們將先探討命定性系統的模糊韌性適應控制，接著再探討非線性隨機系統的模糊韌性適應控制。我們將克服之問題主要有二:

   1)：不假設前提變數之軌跡均落於模糊系統之論域。

   2)：非線性適應控制之廣域穩定度。

   目前於模糊適應控制所發表之論文中，通常藉由假設前提變數之軌跡均落於模糊系統之論域，因而降低了證明適應控制系統穩定度性之困難度，但此假設是相當不合理的，另外於實際應用亦將引起問題。本計劃將突破此問題定為計劃目標。

   　　我們適應控制器設計理念為切換式控制器，依前提變數的位置將控制器設計分為二個情況：前提變數屬於與不屬於模糊系統之論域。當前提變數不屬於模糊系統之論域時，採用 VSS 型態之控制器，使得前提變數回到論域內；當前提變數屬於模糊系統之論域時，採用韌性控制器，使得狀態回到零點。我們利用隨機李亞普諾夫方法，證明韌性適應控制系統的穩定度。

   　　對於命定性一階非線性系統，在不假設前提變數之軌跡均落於模糊系統之論域情況下，本年度計畫已成功地證明切換式適應控制系統之穩定度。未來將朝向命定性高階非線性系統與隨機系統作相關探討。