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Modeling of nonlinear stochastic system and its application to archery - Intended aiming adjustment analysis based on the ARX part of ARMAX model

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摘要

射箭的隨機非線性系統，可分為屬於非線性的瞄射調整，以及具隨機特性肌力穩定方面的。引入T-S模糊模式結合一組線性系統，來表示在放箭前1.5秒時間的瞄射軌跡。因此重要的經驗法則借由線性迴歸移動平均外加輸入的系統來建立。本論文借由對個別及群體射箭員採用統計的分析得到有價值的相關性資料。此資料是排除肌力穩定性只考慮射手主要所想要的瞄射軌跡為主要重點，並借由一種漢米視窗來過濾及排除肌力穩定性的影響，此計畫得到重要且直接影響射箭表現的因素。

關鍵詞：線性迴歸移動、瞄射軌跡、相關性、漢米視窗。

Abstract—A nonlinear stochastic system associated with archery can be separated into two parts, that is, the nonlinear part related to the desired aiming adjustment and the stochastic part associated with the muscle strength stability. The T-S fuzzy model is then adopted to combine the several linear aggressive moving average with an exogenous input (ARMAX) models to represent the nonlinear part. In other words, the ARMAX is adopted to model the aiming trajectory recorded during the last 1.5 second before releasing the arrow. Thus, the important knowledge base for the T-S fuzzy model is attempted to obtain from these linear ARMAX models based on each archer for analyzing his archery performance. Through the statistic correlation approach, the individual and grouping characteristics are obtained from it. During that 1.5 period, the desired adjustments of archers without considering their muscle strength are the main interest in this paper. For expertise archers, their desired aiming style should not contain high frequency which is corresponding negative pole of the model. Therefore, a Hamming window is implemented to remove the high frequency effect resulted from the muscle strength effect. The direct effect on the performance has been found and discussed.

Keywords—ARMAX, aiming trajectory, correlation, Hamming windows

I. INTRODUCTION

Lots of archery researches have been conducted from different approaches in order to find the key point for improving their performance of this fine and highly skilled sport. The most important focus will be the stability of aiming style, so how to use systematic methods to evaluate it falls in the direction. A biomechanical study on the final push-pull archery has been conducted

by Leroyer et al. [1]. The purpose of their study is to analyze archery performance among eight archers of different abilities by means of displacement pull-hand measurements during the final push-pull of the shoot. The archers showed an irregular displacement negatively related to their technical levels. Displacement signal analysis showed high power levels in both 0-5 Hz and 8-12Hz ranges. The latter peak corresponds to electromyographic tremor observed during a prolonged push-pull effort. The results are discussed in relation to some potentially helpful training procedures such as biofeedback and strength conditioning. Landers et al. [2] have examined novice archers to determine whether (a) hemispheric asymmetry and heart rate deceleration occur as a result of learning, and (b) these heart rates and electroencephalograph (EEG) patterns are related to archery performance. The electromyography (EMG) technology which measures the activation patterns in forearm muscles related to contraction and relaxation strategy during archery shooting, has been applied by Ertan et al., [3] to analyze for archers with different levels of expertise; elite, beginner, and non-archers, respectively. They found that elite archers' release started about 100 ms after the fall of the clicker, whereas for beginners and non-archers, their release started after about 200 and 300 ms, respectively. How the novice archers apply the taught training information under different conditions and guided them to promote their motor skills required for better archery performance have investigated by Lavissee et al. [4].

The aiming stability is the key factor affects the archery performance has been indicated by Shiang et al. [5], and it can be determined by the size of aiming locus. They further pointed out that the aiming locus pattern is also a useful index to determine the performance. The effects of heart variable rate (HVR) related to the stability of archer has been measured by C.-T. Lo. [6]. By the frequency-domain analysis of the HVR and three main frequency domains, such as very low frequency (VLF), low frequency (LF), and high frequency (HF). The VLF component is much less defined and the HF generally represents parasympathetic activity. The LF is influenced by both sympathetic and parasympathetic activity, and the ratio of HF to LF represents the balance of parasympathetic and sympathetic activity. The results showed that the HF was higher, the LF was lower, and the LF/HF ratio was lower for the best performance.

Analysis of correlation between the aiming adjustment trajectory and the shot points has been studied [7] by Lin et al. Aiming trajectory can be modeled as a nonlinear stochastic system. The desired aiming adjustment is modeled as the deterministic but nonlinear part, and the muscle strength stability is modeled as the stochastic part. Several linear aggressive moving average with an exogenous input (ARMAX) models to represent the nonlinear part via the T-S fuzzy scheme. Thus, the important knowledge base for the T-S fuzzy model is needed to obtain from these linear ARMAX models. In this paper, ARMAX [8] is adopted to model the aiming trajectory for each shot associated with each archer. We then define related variables for identifying their role upon the performance. The direct effect on the performance related to these variables will be analyzed. The Hamming window is implemented to recovery the desired and anticipated adjustments of archers. Correlation method is designed to obtain the individual and grouping characteristics.

Notations:

TR : the radius distance of the arrow from target center.

X : the horizontal direction.

Y : the vertical direction.

a_x, a_y : the coefficients of AR part of ARMAX model.

b_x, b_y : the exogenous inputs along both directions.

$|bx|, |by|$: the absolute value of the exogenous inputs.

$Px1, Py1$: the first dominate real pole of ARMAX model.

$Px2r, Py2r$: the real part of complex pair pole or the second dominate real pole.

Px_1, Py_1 : the pole is 1.

$Rx1, Ry1$: the corresponding residue of $Px1$ and $Py1$.

$Rx2, Ry2$: the corresponding residue of $Px2$ and $Py2$.

$Rx2r, Ry2r$: the real parts of $Rx2$ and $Ry2$.

$Rx2a, Ry2a$: the absolute values of $Rx2$ and $Ry2$.

Rx_1, Ry_1 : the residue of Px_1 and Py_1 .

$Ux1$: the combined effect of $Px1$ and $Rx1$.

$Uy1$: the combined effect of $Py1$ and $Ry1$.

$Ux2$: the real part of combined effect of $Px2$ and $Rx2$.

$Uy2$: the real part of combined effect of $Py2$ and $Ry2$.

Ux_1 : the combined effect of Px_1 and Rx_1 (the same as Rx_1).

Uy_1 : the combined effect of Py_1 and Ry_1 (the same as Ry_1).

Ux, Uy : the total effect of exogenous inputs b_x and b_y .

UR : $UR = \sqrt{Ux^2 + Uy^2}$

$T-x, T-y$: the settling time of Ux and Uy , respectively.

$T-d$: $|T-x - T-y|$ the absolute value of difference between settling times of $T-x$ and $T-y$.

Ax, Ay : the phase angle of $Px2$ and $Py2$, respectively.

$C(v1, v2)$: the correlation between variables $v1$ and $v2$ based on individual archer.

$CG(v1, v2)$: the correlation between TR and $C(v1, v2)$ based on twelve archers (global sense).

$CGA(v1, v2)$: the correlation between TR and $|C(v1, v2)|$ based on twelve archers (global sense).

II. METHODS

The exerted force related to the intended and desired adjustment to compensate the existing offset (the exogenous part) is modeled as the AR part of the ARMAX. It notes that a negative pole existing at the AR part is corresponding to the high frequency oscillation, so it is not belonged to the desired and intended adjustment. This high frequency oscillation is reasonable to be modeled as the stability of muscle strength. The original recorded data processed by the proposed ARMAX model do obtain the undesired negative pole in the AR part, even though the pole is very close to -1. Therefore, the most common Hamming window is applied to separate the high frequency oscillation (the effect of muscle strength stability) from the AR part which is designed the desired adjustments.

The exogenous part of the ARMAX can be used to describe the main adjustment of the aiming trajectory to compensate the offset between the aiming point and the center of the target. For example, the current aiming point is located at the left of the center of the target, and archers usually will exert a steady force to move the bow right forward the center of the target. This steady constant force is then modeled as a constant b_x . Because of the setting of this model, the right forward constant force is represented by a positive constant. Similarly, the constant b_y is designated for the vertical direction case.

The MA part of the ARMAX is utilized to model the muscle strength of archers. These three coefficients are related to two zeros of the transfer function. Likewise the stability analysis based on them may have connection with the poles of AR part. The mean and variance of the driving noise can indicate the accuracy and fairness of the proposed ARMAX model.

The original aiming trajectory, the smoothing trajectory by the Hamming window, and the estimated trajectory from the previous one are depicted in Fig. 1 for comparison. It is obvious that the muscle strength stability can be separated as the stochastic part, and the ARMAX with order 3 can model the smoothing trajectory well. We now denote the desired aiming trajectories along both directions as Ux and Uy . They are related to the poles of the system and the associated residue, and in the z -domain can be written as

$$Ux(z) = \frac{z^3 b_x}{(z-1)(z^3 - a_{x1}z^2 - a_{x2}z - a_{x3})} = z \left\{ \frac{Rx_1}{(z-1)} + \frac{Rx1}{(z-Px1)} + \frac{Rx2}{(z-Px2)} + \frac{Rx3}{(z-Px3)} \right\} \quad (1)$$

$$Uy(z) = \frac{z^3 b_y}{(z-1)(z^3 - a_{y1}z^2 - a_{y2}z - a_{y3})} = z \left\{ \frac{Ry_1}{(z-1)} + \frac{Ry1}{(z-Py1)} + \frac{Ry2}{(z-Py2)} + \frac{Ry3}{(z-Py3)} \right\} \quad (2)$$

The intended horizontal and vertical adjustments of the exerting force $Ux(k)$ and $Uy(k)$ in the time series,

relating to the offset bx , by , and the AR part of the ARMAX are written as

$$\begin{aligned} U_x(k) &= R_{x_1} + R_{x1}(Px1)^k + R_{x2}(Px2)^k + R_{x3}(Px3)^k \\ &= U_{x_1}(k) + U_{x1}(k) + U_{x2}(k) + U_{x3}(k) \quad (3) \end{aligned}$$

$$\begin{aligned} U_y(k) &= R_{y_1} + R_{y1}(Py1)^k + R_{y2}(Py2)^k + R_{y3}(Py3)^k \\ &= U_{y_1}(k) + U_{y1}(k) + U_{y2}(k) + U_{y3}(k) \quad (4) \end{aligned}$$

Their initial values ($k=0$) are as the same as their associated residue, and their final values ($k=90$) are defined as $U_{x_1} = R_{x_1}$, $U_{x1} = U_{x1}(90)$, $U_{x2} = U_{x2}(90)$, $U_{x3} = U_{x3}(90)$, $U_{y_1} = R_{y_1}$, $U_{y1} = U_{y1}(90)$, $U_{y2} = U_{y2}(90)$, and $U_{y3} = U_{y3}(90)$ for comparing their role playing in the aiming trajectory time series.

The intended horizontal and vertical adjustments of the exerting force along both directions $U_x(k)$ and $U_y(k)$ have been defined, and their initial values and the final values are also evaluated for comparison. Now the physical meanings of them are illustrated by the graph. Three components of the $U_x(k)$, in which the dominate exponential type of adjustment $U_{x1}(k)$, the oscillation type with exponential decay envelope adjustment $U_{x2}(k) + U_{x3}(k)$ and the constant type U_{x_1} are depicted in Fig. 2, respectively. Similarly, those components of the $U_y(k)$, $U_{y2}(k) + U_{y3}(k)$ and U_{y_1} are also shown in Fig. 3. Moreover, the initial values of $U_{x2}(k) + U_{x3}(k)$ is equal to $R_{x2} + R_{x3}$, and in case of the complex pair of $Px2$ and $Px3$, we have $R_{x2} + R_{x3} = 2R_{x2r}$, which is most common in this experiment. The final values of $U_{x1} = U_{x1}(90)$, $U_{x2} + U_{x3} = U_{x2}(90) + U_{x3}(90)$ can be checked at the last points of the graph Fig. 2. The settling time $T_{-x=m}$ is defined that the five consecutive time instances in which $|U_x(k) - \text{mean}(U_x(86\sim 90))| < 0.05 * \text{mean}(U_x(86\sim 90))$, for $k=m, m+1, \dots, m+4$. Accordingly, the longest settling time $m=85$. The same definition is also applied for T_{-y} , so the absolute value of difference between these two settling times is then formulated as $|T_{-x} - T_{-y}|$. The settling time T_{-x} of Fig. 2 is $m=84$ because the dominate pole $Px1=0.999$ is too close to 1. The settling time T_{-y} of Fig. 3 is $m=69$ which is also dominated by the $Py1=0.965$. Usually, the settling time can be dominated by the complex pole pair with a slow exponential decay envelope, that is, the combined effect of the real part of $Px2$ ($Px2r$), its initial value $R_{x2} + R_{x3} = 2R_{x2r}$ and R_{x_1} . Sometimes, the oscillation frequency also plays an important role in the settling time T_{-x} , so the oscillation frequency ($2\pi/T$) is the same as the phase angle of the complex pole pair, so they are defined as follows:

$$\begin{aligned} A_x &= \tan^{-1}(\text{imaginary}(Px2) / Px2r) \\ A_y &= \tan^{-1}(\text{imaginary}(Py2) / Py2r) \end{aligned} \quad (5)$$

The oscillation frequency $A_x=0.608 = 2\pi/10.334$ with the period $T=10.33$ and $A_y=0.422 = 2\pi/14.889$ with the period $T=14.889$ can be observed from Figs. 2 and 3.

III. RESULTS AND DISCUSSION

We note that the variable TR is the most important variable which is directly related to the performance of archers. The sorting method according to the value TR based on twelve shots of each archer is conducted, and the first shot is corresponding to the best shot for individuals. Then the mean of TR of twelve shots of individual archer is calculated as the criterion for categorized these twelve archers, similarly, the archer 1 is the archer with the best performance with the smallest mean of TR . Because the ARMAX model is adopted to model the aiming trajectory associated with each shot and each archer, the essential variables and their physical meanings related to this model are defined and illustrated in previous section.

In this paper, the main objective is to find the crucial effects which are suitable for describing the performance of archers, and to propose some conceivable suggestions for each archer based on group sense or global sense. According to the individual affiliated correlation and its mean of TR , the correlation approach is managed to repeat again to obtain the group or global senses and defined as $CG(v1, v2)$. The results are listed in the lower triangle of Table 1. Since the range of $C(v1, v2)$ is from -1 to 1. If both positive and negative $C(v1, v2)$ have the same effect relative to the key variable TR , then the effect will become unclear by utilizing the $CG(v1, v2)$ approach alone. In order to recover this missing effect, the absolute value of $C(v1, v2)$ relative to the variable TR is defined as $CGA(v1, v2)$. The results of $CGA(v1, v2)$ are also shown in the upper triangle of Table 1. For convenience the threshold 0.45 for correlation coefficients CG and CGA is applied to screen out the strong relationship for further analysis.

Base on the associated correlations CG and CGA , we design an inference algorithm to classify them into several groups. The inference basically is derived from the sufficient condition and necessary condition. Due to the interval of the correlation $[-1, 1]$, if the mean of correlation (C) is greater than zero, then the positive correlation (PC) becomes the sufficient condition and the negative correlation (NC) is the necessary condition. Similarly, for the CGA case the absolute value of C is needed to implement to result in new interval $[0, 1]$, in which the zero is corresponding to low correlation (LC) and the value 1 is associated with high correlation (HC). Since CG and CGA are conducted with TR , the larger and positive CG indicates that the good performance (GP) is related to the more negative correlation, in other words the bad performance (BP) is relative to the more positive correlation. If the mean of correlation is greater than zero, we can say that if the correlation is positive (PC) then the corresponding performance is good (GP). Thus we use the abbreviation PCGP to represent the above inference that is equivalent to the abbreviation BPNC.

In case of $|CG| = |CGA|$, they are four possible cases as outline in the Fig. 4-1. The first case is $CGA > 0$ and $CG > 0$ indicates that the original correlation C is distrib-

uted inside the interval $[0, 1]$, the mean of C is greater than zero, by following the previous inference we can obtain the positive correlation as the dominate sufficient condition which infers PCBP and the equivalent GPLC. The second case ($CGA > 0, CG < 0$) and third case ($CGA < 0, GA > 0$) have the same interval $[-1, 0]$ which is different from the first case and the fourth case ($CGA < 0, GA < 0$). Thereafter, the mean of correlation associated with the second and third cases is less than zero, so we can deduce NCBP (GPLC) for the second case and the deduction NCGP (BPLC) for the third case.

The performance is directly related to the variable TR , so in this section we focus on this particular one. We start with the horizontal axis in which three significant correlations CG or CGA associated with variable $Ux1$, $Px1$, and Ux exist. The last two variables with positive correlations CGA suggest that for the better archer these two variables should have little effect on the performance. For the first variable Ux , related to the $Px1$ and $Rx1$, we have a negative $CG = C(C(TR, Ux1), TR)$ value indicates that the better performance have a positive $C(TR, Ux1)$. The positive $C(TR, Ux1)$ can be further explained that the better performance is, the smaller value $Ux1$ is.

For the vertical case, there is only one significant $CGA(0.52)$ with the variable by , so it suggests that the lower correlation $C(by, TR)$ is good for better performance. The variable by is related to the vertical offset at the 1.5 second instance, so this deduction is reasonable. The good performance of archers should have very consistent adjustments along the vertical direction regardless of the vertical offset. Another three variables $Py1(CG = -0.89)$, $Ay(CG = -0.69)$, and $T-y(CG = -0.56)$ all have negative correlations, so they all indicate that the better performance is connected with positive correlations $C(TR, Py1)$, $C(TR, Ay)$ and $C(TR, T-y)$.

The most significant one is related to $Py1$ which is the dominate pole along the vertical direction, its positive correlation implies that the better performance is associated with the smaller value of pole, in which the exponential decay is fast. This implication is conceivable. Variable Ay is defined as the phase angle of the complex pair $Py2$ and $Py3$, and it is also corresponding to the oscillation frequency of the desired adjustment. So the positive $C(TR, Ay)$ have the physical meaning that the slower oscillation adjustment frequency along the vertical direction can result in better performance. The fast decay of the dominate pole and the slower oscillation adjustment frequency can always result in a fast settling time, so it is confirmed with a positive $C(TR, T-y)$. Because the smaller radius (better performance) and the fast settling time have a positive relationship. The large $T-d$ ($CG = 0.78$) time difference can be easily resulted from the fast settling time $T-y$ along the vertical direction, so this deduction can double confirmed by positive $C(TR, Py1)$, $C(TR, Ay)$ and $C(TR, T-y)$.

The last significant variable $Ry2a$ ($CG = 0.76$) indicates that the negative correlation $C(TR, Ry2a)$ in which the better performance can be achieved by larger $Ry2a$.

This variable is a function of many variables, so it is not easy to explain its effect straight forward. In this section, the direct effect on the performance has been elaborately discussed their physical meanings and their relationship connected to each other.

IV. CONCLUSIONS

The most popular aggressive moving average with an exogenous input (ARMAX) has been adopted and tried to model the aiming trajectories of the twelve archers. It is noted that the recorded trajectories have been processed to represent the last 1.5 second before releasing the arrow. Variables defined from the model are utilized to identify their roles affecting the performance in direct or indirect way through the individual and global statistic correlation approaches. Based on the significant results, some conceivable checking points are suggested for archers to improve their performance. The desired adjustments of archers without affecting by the stability of muscle strength are the main target in this paper. The desired aiming style of expertise should not be a high frequency adjustment. The Hamming window can smooth out the muscle strength effect to obtain their desired one.

A simple inference based on the sufficient and necessary conditions principle has been proposed to link these variables and the performance. Variables have the most important direct effect on the performance have been analyzed as much as possible. Variable Ux , related to the $Px1$ and $Rx1$, has a negative CG value indicates that the better performance have a positive $C(TR, Ux1)$. The positive $C(TR, Ux1)$ further suggests that the better performance is relative to the smaller value $Ux1$.

For the vertical case, there is only one significant $CGA(0.52)$ with the variable by , so it suggests that the lower correlation $C(by, TR)$ is good for better performance. Archers with good performance are supposed to have very consistent adjustments along the vertical direction regardless of the vertical offset. The dominate pole $Py1$ has positive correlation with TR implies that the better performance is associated with the smaller value of pole or fast exponential decay. We have deduced that the slower oscillation adjustment frequency Ay is directly related to better performance. The fast decay of the dominate pole and the slower oscillation adjustment frequency are also linked to a fast settling time $T-y$. Moreover, the large $T-d$ ($CG = 0.78$) time difference can be easily resulted from the fast settling time $T-y$, so this deduction can double confirmed by positive $C(TR, Py1)$, $C(TR, Ay)$ and $C(TR, T-y)$.

This project has created the following papers:

APCST 2007	Archery performance analysis based on the coefficients of AR2 model of aiming trajectory
APCST2007	MATHEMATICAL MODEL OF THE AIMING TRAJECTORY
2008 ICMLC International Conference on Machine Learning and Cybernetics	AIMING TRAJECTORY ANALYSIS BASED ON ARMAX MODEL

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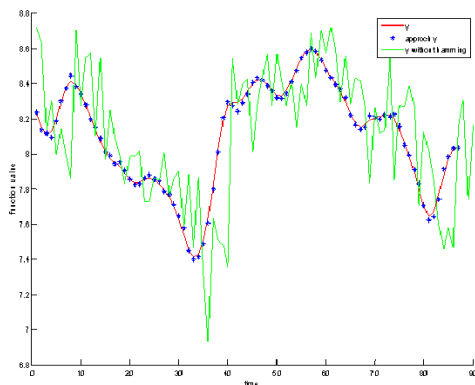


Figure 1: Comparison among the original aiming trajectory, trajectory smoothing by the Hamming window and the estimated trajectory

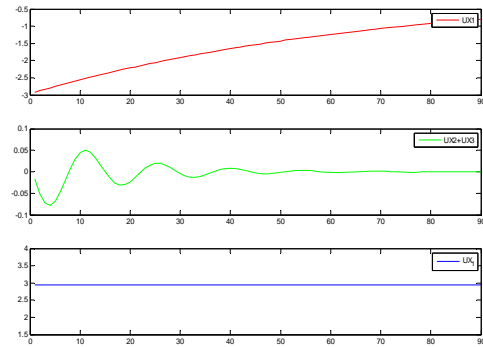


Figure 2: Three components of the intended adjustment U_x along the horizontal direction.

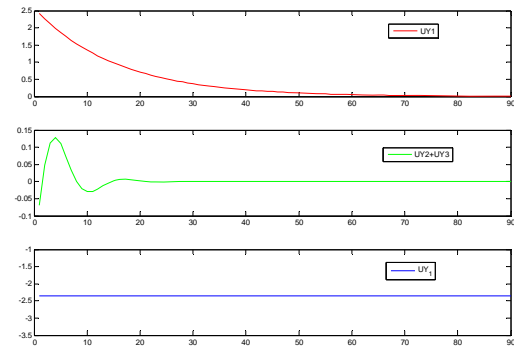


Figure 3 Three components of the intended adjustment U_y along the vertical direction.

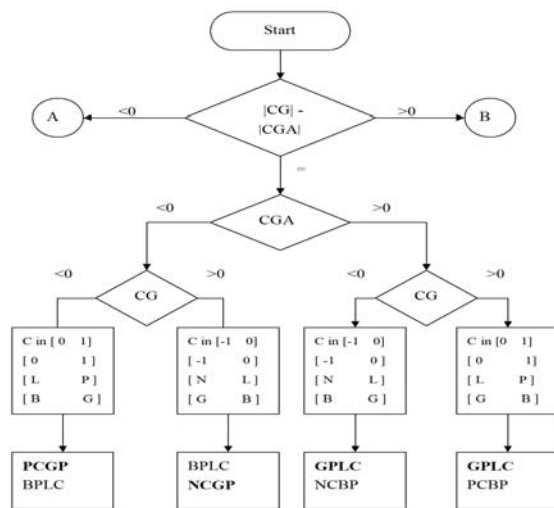


Figure 4-1: Performance Inference Procedure 1

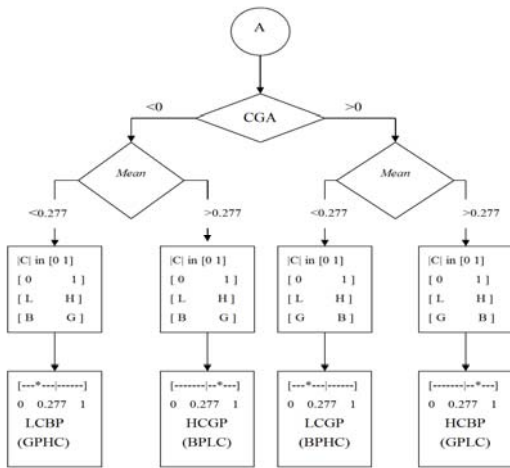


Figure 4-2: Performance Inference Procedure 2

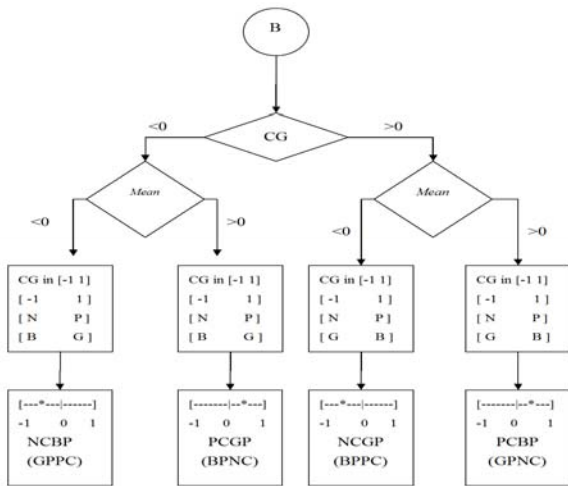


Figure 4-3: Performance Inference Procedure 3

