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利用全向輪推動之球輪機器人機構與控制設計
研究成果報告(精簡版)

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Abstract

Model of the spherical robot driven by Omni wheels and its constant speed control have been derived and written as two conference papers published at the ICMLC2011. The modeling is derived based on the Euler Lagrange approach. The constant speed control is implemented under the sliding mode control of the variable structure control. Current works are focused on the hierarchic SMC (HSMC) and cascade SMC (CSMC). To overcome the constant speed problem which is caused by the fast body attitude convergent rate, that is, vertical attitude, the state switching scheme of both controls has been modified as a periodic switching scheme which releasing the body attitude convergent rate periodically. Spherical wheel position and body attitude controls can share a non-strictly convergence under the periodic releasing feature. Simulations show that the position control of the under-actuated spherical robot can be easily implemented under periodic switching scheme.

Keywords: Modeling; spherical robot; Omni wheels; Euler Lagrange; Variable Structure Control; Sliding mode control; Hierarchic SMC; Cascade SMC.

摘要

關於全向輪驅動的球型機器人的數學模型和定速控制法則已被導出，且已發表兩份研討討會論文於 ICMLC2011。模型由尤拉−拉格朗日的方法推導出，定速控制則由可變結構控制下的滑模控制來實現。目前正在進行分層滑模控制及串聯滑模控制，然而這兩種滑模控制容易造成主體姿態部分收斂太快為直立，進而造成球的定速問題。修改這兩種的狀態切換法則為週期性的，主要是放鬆主體姿態的嚴格收斂速度，避免球的定速問題。換言之，球輪的位置控制與主體姿態控制，在週期放鬆特性下可享有非嚴格收斂的機會。最後在模擬中顯示，欠驅動的球型機器人的位置控制，可以容易地在週期切換法則下實現。

關鍵字：數學模式、球輪機器人、全向輪、尤拉−拉格朗日、可變結構控制、滑模控制、分層滑模控制、串聯滑模控制。

前言　In recent years, researches on simple and small structure mobile robots which can easily be carried or transferred for applying to various aspects of usage in many constrained environments become more and more popular. Other than the weight and structure of a mobile robot, the performance such as vehicle body balance, stability control, speed and positioning control is also one of main factors to determine the applications of that robot.

研究目的　This paper mainly outlines the model of the invented spherical robot using Omni wheels to drive a spherical wheel. The dynamical model is derived based on Euler Lagrange approach. Therefore, based on the derived model, the variable structure control (VSC) is presented in which the sliding mode control (SMC) is adopted to achieve a constant speed at a vertical balance altitude. Simulations of the proposed control algorithm have been conducted based on two pre-determined sliding surfaces with adjustable parameters to discuss the effective time to enter the sliding surface and the convergence.

文獻探討　In 1994, a two-wheeled robot has been proposed [1], and the stability and tracking control of the two-wheeled robot are similar to the use of inverted pendulum control. The most common application of two-wheeled vehicles functioned as the inverted pendulum robot is Segway. It is a very good invention to be studied based on suitable sensors. A precision gyroscope and a sensitive tilt sensor are mainly used in the Segway vehicle to measure data which help to adjust the future road conditions in the different stability of walk [2].

Thereafter, a single wheel with inverse mouse-ball drive can achieve the static and dynamic stability has been developed by Carnegie Mellon University (CMU) [3-4]. The overall design of the system, such as actuator mechanism and control system is presented. Performance of dynamic balancing, station keeping, and point-to-point motion are also discussed and presented. Most of all, their papers pointed out that unlike balancing 2-wheel platforms which must turn before driving in any direction, and the single-wheel can move directly in any directions. Therefore, they are the first group to propose a balancing rolling machine whose body is supported by a single Omni-directional spherical wheel. However, for the CMU robot, the conflict demand of both a high-friction and low-friction material at the same time for the spherical ball becomes the serious concern to be compromised. The novel combination of Omni wheel and spherical wheel (CWWU) has been proposed in [5-6], and it virtually can be expressed as the mobile robot body installing on the spherical wheel which is driven and controlled by two perpendicular pairs of Omni wheels. Both mobile robots with similar structure, the control of the CWWU is also equivalent that of CMU.

The evolution of variable structure control (VSC) is a very popular and powerful control algorithm, and it is a form of discontinuous nonlinear control [7-8]. The algorithm adopts a high-frequency switching control to alter the dynamics of a nonlinear system. Therefore, its state-feedback control law is not a continuous function of time; it switches from one smooth condition to another. So the position of the state trajectory determines the structure of the control law. VSC and associated sliding mode behavior were first investigated by Emelyanov and several co-researchers in early 1950s in the Soviet Union [9]. Recently, the sliding mode control (SMC) is the main operation of VSC [10]. Due to discontinuous control law, its features include low sensitivity to the associated uncertainty of plant parameter, greatly reduced-order modeling of plant dynamics, and finite-time convergence. But, the chattering caused by the implementation imperfections and over-focus on matched uncertainties are its weaknesses.

This paper mainly concentrates on the CWWU, so its dynamical model is presented first based on Euler Lagrange approach [11-12]. The SMC of VSC with two selected sliding surfaces for two axes will be designed and implemented based on the derived dynamics model, and Matlab simulations are also presented.

研究方法

1. System description

In order to derive the dynamical model of the proposed CWWU, we begin with the position $\vec{P}_{sw}$ and velocity $\vec{v}_{sw}$ of the spherical wheel formulated as equations (1) and (2). Thus, its kinetic energy K_{sw} can be written as equation (3)

$$\vec{P}_{sw} = R\phi_1 \vec{i} + R\phi_2 \vec{j} \tag{1}$$

$$\vec{v}_{sw} = R\phi_1' \vec{i} + R\phi_2' \vec{j} \tag{2}$$

$$K_{sw} = \frac{1}{2}[I_{sw}(\phi_1'^2 + \phi_2'^2) + m_{sw}v_{sw}^2] = \frac{1}{2}(I_{sw} + m_{sw}R^2)(\phi_1'^2 + \phi_2'^2) \tag{3}$$

where R is the radius; m_{sw} and I_{sw} are the mass and the moment of inertia; ϕ_1 and ϕ_2 are the rotating angles along the first and second directions; ϕ_1' as well as ϕ_2' are the angular velocities. Here, we do not consider its potential energy because of its invariance.

Similarly, for driving wheels, r is the radius; I_{dw} is the moment of inertia; $R\phi_1 / r$ and $R\phi_2 / r$ are the rotating angles. The exerting torques τ_1, τ_2 of driving wheels will be expressed as the effective ones for the spherical wheel and the body as $(R/r)^2\tau_1$, $(R/r)^2\tau_2$. Because the total mass of driving wheels will be considered as part of body mass, its translating energy and potential energy will be included in the body. Here, only its rotating kinetic energy is considered.

$$K_{dw} = \frac{1}{2}(I_{dw1}(\frac{R}{r})^2\phi_1'^2 + I_{dw1}(\frac{R}{r})^2\phi_2'^2) = \frac{1}{2}(I_{d1}\phi_1'^2 + I_{d1}\phi_2'^2) \tag{4}$$

where $I_{d1} = I_{dw1}(R/r)^2$ and $I_{d2} = I_{dw2}(R/r)^2$. For the body with angles θ_1 and θ_2, and mass m_b centered at a distance ℓ from the center of spherical wheel, then its vertical position of the mass center can be derived as $R + \ell\sqrt{1 - S_1^2 - S_2^2}$, where $S_1 \triangleq \sin\theta_1$ and $S_2 \triangleq \sin\theta_2$, as shown in Figure 1.

As mentioned before the mass m_b also includes the weights of the driving wheels, so the translation kinetic energy and potential energy of driving wheels are taken account into the body. So its position $\vec{P}_b$ and velocity $\vec{v}_b$ are written as equations (5) and (6). Thus, its potential energy U and kinetic energy K_b can be written as equations (7) and (8).

$$\vec{P}_b = (R\phi_1 - \ell S_1)\vec{i} + (R\phi_2 - \ell S_2)\vec{j} + (R + \ell\sqrt{1 - S_1^2 - S_2^2})\vec{k} \tag{5}$$

$$\vec{v}_b = (R\phi_1' - \ell C_1\omega_1)\vec{i} + (R\phi_2' - \ell C_2\omega_2)\vec{j} + \ell\frac{-S_1C_1\omega_1 - S_2C_2\omega_2}{\sqrt{1 - S_1^2 - S_2^2}}\vec{k} \tag{6}$$

$$U = m_b g\ell\sqrt{1 - S_1^2 - S_2^2} \tag{7}$$

$$K_b = \frac{1}{2}[I_x\omega_1^2 + I_y\omega_2^2 - 2I_{xy}\omega_1\omega_2 + m_b v_b^2] \tag{8}$$

where $C_1 \triangleq \cos\theta_1$ and $C_2 \triangleq \cos\theta_2$; I_x, I_y, and I_{xy} are the moment inertia of the body; ω_1 and ω_2 are the angular velocities of the body along both directions; g is the gravity acceleration. After substituting equation (6) into equation (8), the K_b can be written as follows:

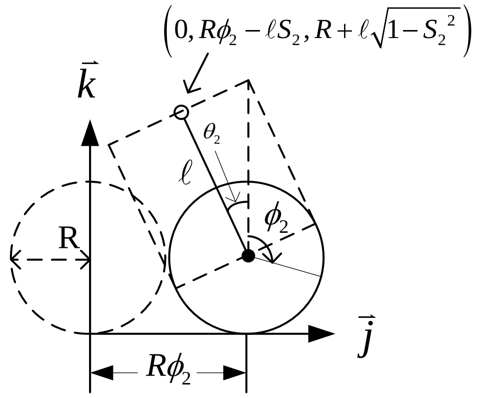

Figure 1. Coordinates of the spherical robot

$$K_b = \frac{1}{2}[I_x + m_b \ell^2 \frac{C_1^{\,2} C_2^{\,2}}{1-S_1^{\,2}-S_2^{\,2}}]\omega_1^2 + \frac{1}{2}[I_y + m_b \ell^2 \frac{C_1^{\,2} C_2^{\,2}}{1-S_1^{\,2}-S_2^{\,2}}]\omega_2^2$$
$$+[m_b \ell^2 \frac{S_1 C_1 S_2 C_2}{1-S_1^{\,2}-S_2^{\,2}} - I_{xy}]\omega_1\omega_2 + \frac{1}{2} m_b R^2 (\phi_1'^2 + \phi_2'^2) - m_b R \ell C_1 \phi_1' \omega_1 - m_b R \ell C_2 \phi_2' \omega_2. \tag{9}$$

We now define the Euler-Lagrange variable $L = K - U = K_{sw} + K_b + K_{dw} - U$, and it can be arranged as equation (10).

$$L = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 - I_{12}\omega_1\omega_2 + \frac{1}{2} J_1 \phi_1'^2 + \frac{1}{2} J_2 \phi_2'^2 - N_1 \phi_1' \omega_1 - N_2 \phi_2' \omega_2 - U \tag{10}$$

where $I_1 = I_x + m_b \ell^2 \frac{C_1^{\,2} C_2^{\,2}}{1-S_1^{\,2}-S_2^{\,2}};\ I_2 = I_y + m_b \ell^2 \frac{C_1^{\,2} C_2^{\,2}}{1-S_1^{\,2}-S_2^{\,2}};\ I_{12} = I_{xy} - m_b \ell^2 \frac{S_1 C_1 S_2 C_2}{1-S_1^{\,2}-S_2^{\,2}}$

$J_1 = I_{sw} + (m_{sw} + m_b)R^2 + I_{d1};\ J_2 = I_{sw} + (m_{sw} + m_b)R^2 + I_{d2};\ N_1 = m_b R \ell C_1;\ N_2 = m_b R \ell C_2$

After formulating the variable of Euler-Lagrange, the dynamical model can be derived from

$$\frac{d}{dt}\frac{\partial L}{\partial q'} - \frac{\partial L}{\partial q} = \tau. \tag{11}$$

where q will correspond to $\theta_1, \theta_2, \phi_1$, and ϕ_2, and its derivative is q', that is, $\omega_1, \omega_2, \phi_1'$, and ϕ_2'.

For the θ_1 and ω_1, based on equations (10) and (11) with $\omega_1' = \alpha_1$, we have

$$I_1\alpha_1 - I_{12}\alpha_2 + \frac{1}{2}\frac{\partial I_1}{\partial \theta_1}\omega_1^{\,2} + \frac{\partial I_1}{\partial \theta_2}\omega_1\omega_2 - (\frac{\partial I_{12}}{\partial \theta_2} + \frac{1}{2}\frac{\partial I_2}{\partial \theta_1})\omega_2^{\,2} + \frac{\partial U}{\partial \theta_1} = -\frac{R}{r}\tau_1 \tag{12}$$

Similarly, for the θ_2 and ω_2, with $\omega_2' = \alpha_2$, we also can get

$$I_2\alpha_2 - I_{12}\alpha_1 + \frac{1}{2}\frac{\partial I_2}{\partial \theta_2}\omega_2^{\,2} + \frac{\partial I_2}{\partial \theta_1}\omega_1\omega_2 - (\frac{\partial I_{12}}{\partial \theta_1} + \frac{1}{2}\frac{\partial I_1}{\partial \theta_2})\omega_1^{\,2} + \frac{\partial U}{\partial \theta_2} = -\frac{R}{r}\tau_2 \tag{13}$$

Finally, for the spherical wheel, we obtain

$$J_1\phi_1'' - N_1\alpha_1 - \frac{\partial N_1}{\partial \theta_1}\omega_1^{\,2} = \frac{R}{r}\tau_1; \quad J_2\phi_2'' - N_2\alpha_2 - \frac{\partial N_2}{\partial \theta_2}\omega_2^{\,2} = \frac{R}{r}\tau_2 \tag{14}$$

Equations (12-13) can be rewritten as the following simplified form, in which $\left[(R/r)\tau_1 \quad (R/r)\tau_2\right]^T$ becomes the virtual control input term to control the exerting torque of driving wheels to be vertical attitude.

$$M\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}+\Pi\begin{bmatrix}\omega_1{}^2\\ \omega_1\omega_2\\ \omega_2{}^2\end{bmatrix}+\begin{bmatrix}\dfrac{\partial U}{\partial\theta_1}\\ \dfrac{\partial U}{\partial\theta_2}\end{bmatrix}=\begin{bmatrix}\dfrac{-R}{r}\tau_1\\ \dfrac{-R}{r}\tau_2\end{bmatrix} \tag{15}$$

where $M=\begin{bmatrix}I_1 & -I_{12}\\ -I_{12} & I_2\end{bmatrix};\Pi=\begin{bmatrix}\dfrac{1}{2}\dfrac{\partial I_1}{\partial\theta_1} & \dfrac{\partial I_1}{\partial\theta_2} & -(\dfrac{1}{2}\dfrac{\partial I_2}{\partial\theta_1}+\dfrac{\partial I_{12}}{\partial\theta_2})\\ -(\dfrac{1}{2}\dfrac{\partial I_1}{\partial\theta_2}+\dfrac{\partial I_{12}}{\partial\theta_1}) & \dfrac{\partial I_2}{\partial\theta_1} & \dfrac{1}{2}\dfrac{\partial I_2}{\partial\theta_2}\end{bmatrix}$

By cancelling both of torques, that is, substituting equation (14) into equation (15), we can have the simplified form as equation (16). It can be found that $\left[J_1\phi_1'' \quad J_2\phi_2''\right]^T$ becomes the virtual control input term for controlling the body to be vertical attitude.

$$M_2\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}-\Pi_2\begin{bmatrix}\omega_1{}^2\\ \omega_1\omega_2\\ \omega_2{}^2\end{bmatrix}+\begin{bmatrix}\dfrac{\partial U}{\partial\theta_1}\\ \dfrac{\partial U}{\partial\theta_2}\end{bmatrix}=-\begin{bmatrix}J_1\phi_1''\\ J_2\phi_2''\end{bmatrix} \tag{16}$$

where $M_2=\begin{bmatrix}I_1-N_1 & -I_{12}\\ -I_{12} & I_2-N_2\end{bmatrix};\Pi_2=\begin{bmatrix}\dfrac{1}{2}\dfrac{\partial I_1}{\partial\theta_1}-\dfrac{\partial N_1}{\partial\theta_1} & \dfrac{\partial I_1}{\partial\theta_2} & -(\dfrac{1}{2}\dfrac{\partial I_2}{\partial\theta_1}+\dfrac{\partial I_{12}}{\partial\theta_2})\\ -(\dfrac{1}{2}\dfrac{\partial I_1}{\partial\theta_2}+\dfrac{\partial I_{12}}{\partial\theta_1}) & \dfrac{\partial I_2}{\partial\theta_1} & \dfrac{1}{2}\dfrac{\partial I_2}{\partial\theta_2}-\dfrac{\partial N_2}{\partial\theta_2}\end{bmatrix}$

The associated partial derivatives are summarized as below.

$$\partial N_1/\partial\theta_1=-m_bR\ell S_1;\ \partial N_2/\partial\theta_2=-m_bR\ell S_2;\partial I_1/\partial\theta_2=\partial I_2/\partial\theta_2=2m_b\ell^2\frac{S_1{}^2C_1{}^2C_2S_2}{(1-S_1{}^2-S_2{}^2)^2};$$

$$\partial I_1/\partial\theta_2=\partial I_2/\partial\theta_2=2m_b\ell^2\frac{S_1{}^2C_1{}^2C_2S_2}{(1-S_1{}^2-S_2{}^2)^2};\partial I_{12}/\partial\theta_1=-m_b\ell^2S_2C_2\frac{C_1{}^2C_2{}^2+S_1{}^2S_2{}^2}{(1-S_1{}^2-S_2{}^2)^2};$$

$$\partial I_{12}/\partial\theta_2=-m_b\ell^2S_1C_1\frac{C_1{}^2C_2{}^2+S_1{}^2S_2{}^2}{(1-S_1{}^2-S_2{}^2)^2}.$$

2. SMC of VSC

In the section, the SMC of VSC for the spherical robot is proposed. Two sliding surfaces are designed along both directions for reducing the dimension of the system. The convergence of the body attitude can be adjusted by two designing positive real parameters a_1 and a_2. A positive Lyapunov function or cost function is selected as

$$L_{VSS}=\frac{1}{2}(\omega_1+a_1\theta_1)^2+\frac{1}{2}(\omega_2+a_2\theta_2)^2. \tag{17}$$

Then, its derivative is

$$\frac{dL_{VSS}}{dt}=(\omega_1+a_1\theta_1)(\alpha_1+a_1\omega_1)+(\omega_2+a_2\theta_2)(\alpha_2+a_2\omega_2). \tag{18}$$

By designing

$$\begin{aligned}\alpha_1+a_1\omega_1&=-sign(\omega_1+a_1\theta_1)(\alpha_1+a_1\omega_1)^{2n},\\ \alpha_2+a_2\omega_2&=-sign(\omega_2+a_2\theta_2)(\alpha_2+a_2\omega_2)^{2n},\end{aligned} \tag{19}$$

where parameter n is a non-negative integer, we have a negative derivative of the cost function

$$\begin{aligned}\frac{dV}{dt} &= -sign(\omega_1 + a_1\theta_1)(\omega_1 + a_1\theta_1)^{2n+1} - sign(\omega_2 + a_2\theta_2)(\omega_2 + a_2\theta_2)^{2n+1} \\ &= -\left|(\omega_1 + a_1\theta_1)\right|^{2n+1} - \left|(\omega_2 + a_2\theta_2)\right|^{2n+1} \le 0.\end{aligned} \tag{20}$$

Therefore, the virtual control law can be written as

$$\begin{bmatrix} J_1\phi_1'' \\ J_2\phi_2'' \end{bmatrix} = M_2 \begin{bmatrix} sign(\omega_1 + a_1\theta_1)(\omega_1 + a_1\theta_1)^{2n} + a_1\omega_1 \\ sign(\omega_2 + a_1\theta_2)(\omega_2 + a_2\theta_2)^{2n} + a_2\omega_2 \end{bmatrix} + \prod\nolimits_2 \begin{bmatrix} {\omega_1}^2 \\ \omega_1\omega_2 \\ {\omega_2}^2 \end{bmatrix} - \begin{bmatrix} \dfrac{\partial U}{\partial \theta_1} \\ \dfrac{\partial U}{\partial \theta_2} \end{bmatrix}. \tag{21}$$

When the virtual controls $[J_1\phi_1''\ J_2\phi_2'']^T$ become zero, it implies that the spherical robot can only reach the constant speed, that is, $\phi'' = 0 \to \phi'$ is constant. In the section, convergence of body attitude can be guaranteed by observing the negative derivative of the selected positive cost function as equation (20). Equation (21) is the derived SMC of VSC for the spherical robot, and three parameters n, a_1, and a_2 can be designed for different considerations or applications. In the next section, simulations will be implemented for realizing the effect of parameters.

3. Simulations and Discussions

In order to test the performance of the proposed control laws, we carry out several numerical simulations using MATLAB™. In these simulations, the spherical wheel is assumed to be made as a hollow sphere with radius $R = 0.1m$ and mass $m_{sw} = 0.25kg$; the driving wheels are the thin solid disks, each one with the same radius $r = 0.1m$ and the same mass $m_{dw} = 0.2kg$; the body is a solid cylinder in which the radius $R_b = 0.1m$, mass $m_b = 7kg$ and height $h = 2\ell = 0.4m$.

Table 1: Various setting of parameters

Case	Parameters		
	Sub-case	n	$a_1 = a_2$
Case_	I_1	0	1
1	I_2	0	5
Case_	II_1	0	2
2	II_2	1	2

In simulations, the initial attitudes of body are $\theta_1(0) = -\pi/12$ and $\theta_2(0) = \pi/6$, with angular velocities $\omega_1(0) = \omega_2(0) = 0$; the angles and angular velocities of spherical wheel are all zeros, that is, $\phi_1(0) = \phi_1'(0) = 0$ and $\phi_2(0) = \phi_2'(0) = 0$. In order to compare the effect of various setting of parameters $a_1 = a_2$ and n, as listed in Table 1, the normalization of the Lyapunov is needed, that is, for each simulated case, it always starts from one.

The Lyapunov function is decreasing as expected for all simulations, as observed from Figure 2. For the same setting of parameter $n = 0$, the larger value of parameter a_1 (CaseI_2 > CaseII_1 >CaseI_1) will cause the slower convergent rate of the cost function. Moreover, Figure 3 indicates that parameter a_1 is larger, and the time to enter the designed sliding surface will be longer. This implies that for larger value of parameter a_1

can result in a shorter period of switching control inputs or longer period of smoothing control inputs. Moreover, Figure 7 indicates that the steady-state constant speed of the spherical wheel depends on the setting of parameter a_1, and the better one in the simulation is $a_1 = 2$ when $n = 0$.

The larger value of parameter n can result in slower convergence of the cost function during the latter portion, as referred to Figure 2, and its trajectory will take very long period to reach the sliding surface or the zero tilt attitude, as shown in Figure 4. It even cannot reach the constant speed of spherical wheel in time or reach higher constant speed after convergence, as illustrated from Figures 5 and 6, This scenario can be illustrated by equation (20) in which the derivative of the cost functional will be more negative when $|(\omega_1 + a_1\theta_1)|$ or $|(\omega_2 + a_2\theta_2)|$ greater than one. In the other hand, the derivative of the cost functional will be less negative when $|(\omega_1 + a_1\theta_1)|$ or $|(\omega_2 + a_2\theta_2)|$ less than one.

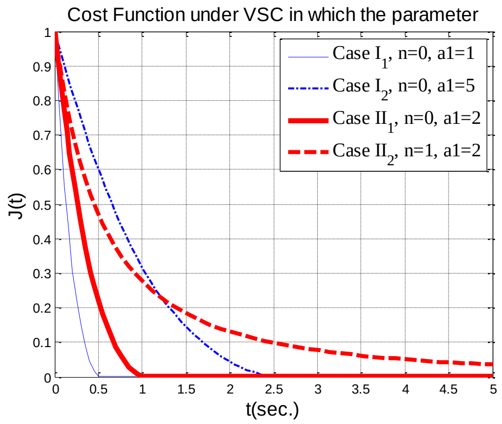

Figure 2: Lyapunov functions of the proposed VSC.

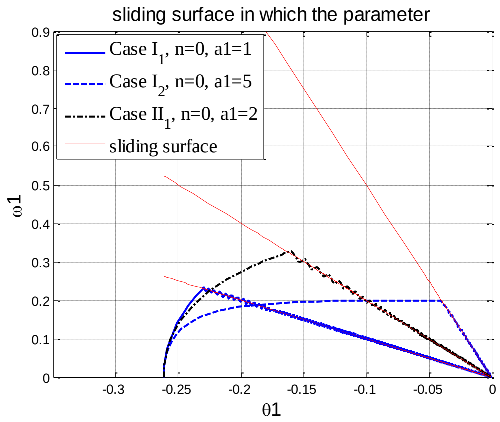

Figure 3: Trajectories of the sliding surface

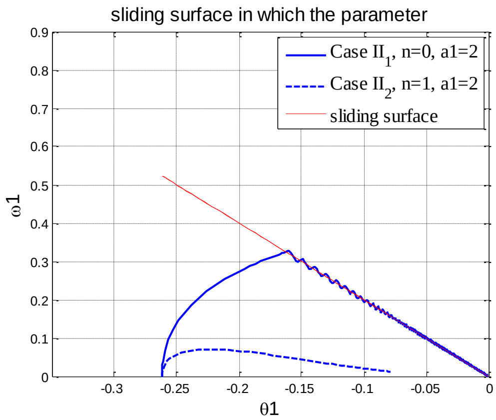

Figure 4: Trajectories of the sliding surface

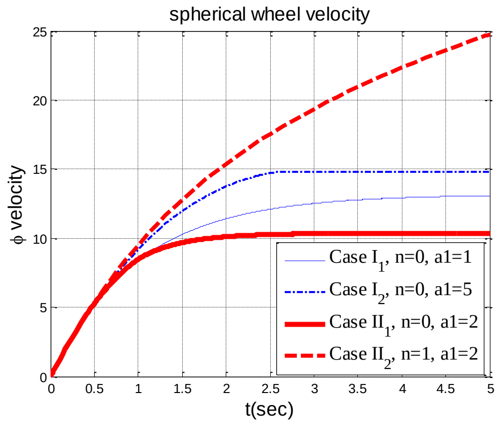

Figure 5: Spherical angular velocity for all cases.

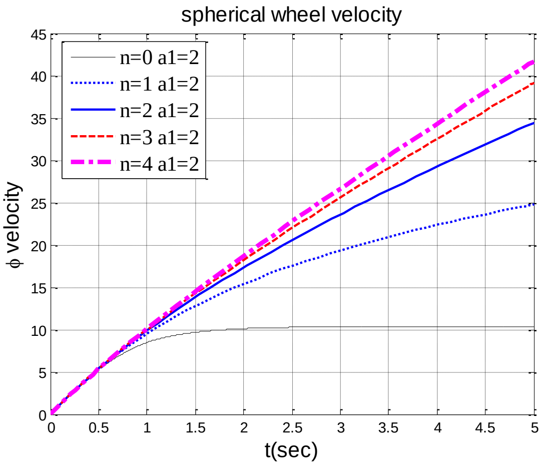

Figure 6: Spherical angular velocity.

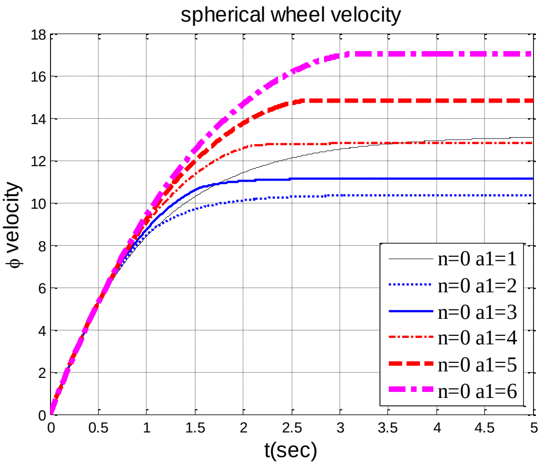

Figure 7: Spherical angular velocity.

4. Conclusions

In this paper, the model of the spherical wheel is outlined which is different from the previous researches. Therefore, based on the new derived model, the constant speed of the spherical robot with the SMC of the VSC has been derived and simulated. The effect of two designable parameters, such as, the convergence on the sliding surface and the steady-state speed of the spherical robot, has also been studied and discussed for understanding their role in the proposed SMC.

Acknowledgements

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Current Work:

The modeling and constant speed control of the proposed spherical robot have been published at ICMLC2011 [1-2]. Its position control will focus on powerful hierarchic SMC (HSMC) and cascade SMC (CSMC) for under-actuated systems. However, these applications to the proposed spherical robot will easily result in undesired constant velocity due to the fast convergence of the body of robot. Therefore, we propose a periodic switching scheme instead of state switching scheme, that is, periodic hierarchic SMC (PHSMC) and periodic cascade SMC (PCSMC).

Consider a two dimensional under-actuated system.

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = f_1 + b_{11}u_1 + b_{12}u_2\\ \dot{x}_3 = x_4\\ \dot{x}_4 = f_2 + b_{21}u_1 + b_{22}u_2\end{cases} \quad \text{and} \quad \begin{cases}\dot{x}_5 = x_6\\ \dot{x}_6 = f_3 + b_{31}u_1 + b_{32}u_2\\ \dot{x}_7 = x_8\\ \dot{x}_8 = f_4 + b_{41}u_1 + b_{42}u_2\end{cases}$$

We design four sliding surfaces $s_1 = c_1x_1 + c_2x_1^{n_1} + x_2$, $s_2 = c_3x_3 + c_4x_3^{n_2} + x_4$, $s_3 = c_5x_5 + c_6x_5^{n_3} + x_6$, and $s_4 = c_7x_7 + c_8x_7^{n_4} + x_8$. The hierarchic combination of these four sliding surfaces as $S_A = k_1s_1 + k_2s_2$ and $S_B = k_3s_3 + k_4s_4$. For the cost function $V = V_A + V_B = \frac{1}{2}S_A^2 + \frac{1}{2}S_B^2$, its individual derivative by letting

$$u_1 = u_{eq1} + u_{eq2} + u_{sw1} = \frac{k_1\left(-c_1x_2 - f_1 - n_1c_2x_1^{n_1-1}x_2\right)}{\left(k_2b_{21} + k_1b_{11}\right)} + \frac{k_2\left(-c_3x_4 - f_2 - n_2c_4x_3^{n_2-1}x_4\right)}{\left(k_2b_{21} + k_1b_{11}\right)} - \frac{\eta sign(S_A) + kS_A}{\left(k_2b_{21} + k_1b_{11}\right)},$$

can be written as:

$$\dot{V}_A = S_A\dot{S}_A = S_A\left[k_1\dot{s}_1 + k_2\dot{s}_2\right]$$
$$= S_A\left[k_1\left(c_1x_2 + f_1 + b_{11}u_1 + n_1c_2x_1^{n_1-1}x_2\right) + k_2\left(c_3x_4 + f_2 + b_{21}u_1 + n_2c_4x_3^{n_2-1}x_4\right)\right]$$
$$= S_A\left\{k_1\left(c_1x_2 + f_1 + b_{11}\left(u_{eq1} + u_{eq2} + u_{sw1}\right) + n_1c_2x_1^{n_1-1}x_2\right) + k_2\left(c_3x_4 + f_2 + b_{21}\left(u_{eq1} + u_{eq2} + u_{sw1}\right) + n_2c_4x_3^{n_2-1}x_4\right)\right\}$$
$$= S_A\left[-\eta sign(S_A) - kS_A\right]$$

Similarly, we obtain $\dot{V}_B = S_B\left[-\eta sign(S_B) - kS_B\right]$ by designing

$$u_2 = u_{eq3} + u_{eq4} + u_{sw2} = \frac{k_3\left(-c_5x_6 - f_3 - n_3c_6{x_5}^{n_3-1}x_6\right)}{\left(k_2b_{21} + k_1b_{11}\right)} + \frac{k_4\left(-c_7x_8 - f_4 - n_4c_8{x_7}^{n_4-1}x_8\right)}{\left(k_2b_{21} + k_1b_{11}\right)} - \frac{\eta sign(S_B) + kS_B}{\left(k_2b_{21} + k_1b_{11}\right)}.$$

The conventional hierarchic SMC (HSMC) is to set parameters as $n_j = 0\ \forall\, j, k_1 = k_3 = 10$, $c_4 = c_8 = 0$ and its key state switching scheme is that $k_2 = sign(s_1s_2)$ and $k_4 = sign(s_3s_4)$. The main idea of the state switching is to avoid the following two possibilities: (1) $s_1 \neq 0$ or $s_2 \neq 0$ but $S_A = k_1s_1 + k_2s_2 = 0$, (2) $s_3 \neq 0$ or $s_4 \neq 0$ but $S_B = k_3s_3 + k_4s_4 = 0$. There is a special phenomenon that the zero vertical body attitude of spherical robot will cause the difficulty to move the spherical ball to the desired position. The conservative state scheme will result in the fast convergence of the spherical robot body to result in the constant speed of the spherical ball.

Therefore, we propose the following periodic switching scheme to overcome the undesired constant speed drawback by releasing the fast convergence of the body periodically:

$$k_2(t) = k_2(t+2T) = \begin{cases} 1 & 0 < t \le T \\ -1 & T < t \le 2T \end{cases} \quad \text{and} \quad k_4(t) = k_4(t+2T) = \begin{cases} 1 & 0 < t \le T \\ -1 & T < t \le 2T \end{cases}.$$

The parameter setting PHSMC is as the same as those of HSMC, that is, $n_j = 0\ \forall\, j, k_1 = k_3 = 10$. The periodic releasing scheme sometimes may cause the instability of the body. In order to reduce the possibility of the instability of the body, we propose a new PHSMC as PHSMC1 by setting $n_1 = n_3 = 3$. For the HSMC1, the body can converge fast when current angle is greater than a predetermined angle, and on the other hand, the body converges slower when current angle is less than the predetermined angle. The predetermined angles of both dimension are dependent on the constants c_2 or c_6. The larger values of these constants are, the smaller predetermined angles are.

For comparison of HSMC, PHSMC, and PHSMC1, simulations have been carried out with the following setting: $2T = 1/50\,\text{sec}$, $c_2 = c_6 = 180/5\pi$, $k = 1, \eta = 0, c_1 = c_5 = 2, c_3 = c_7 = 1$. Figures 1-1 and 1-3 show that the angular position and velocity of the body of both dimensions converge to zero for all controls. It notes that the convergent rate is fastest for the HSMC as expected. Figures 1-2 and 1-4 indicate the undesired constant speed of the spherical wheel for the HSMC. Therefore, the proposed PHSMC and PHSMC1 can solve the constant speed problem to achieve the position control for the under-actuated spherical robot.

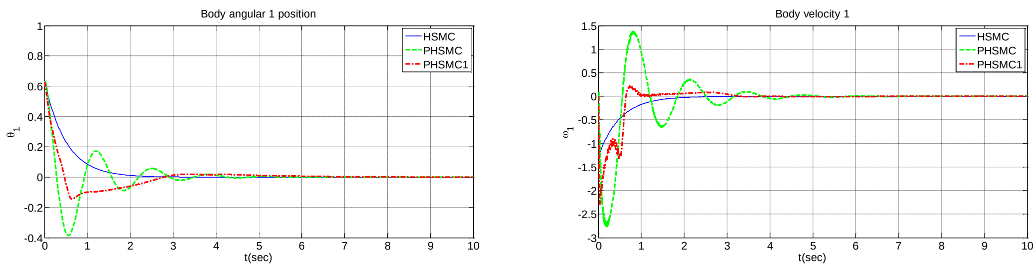

Figure 1-1 Angular position and velocity of the body of the first dimension

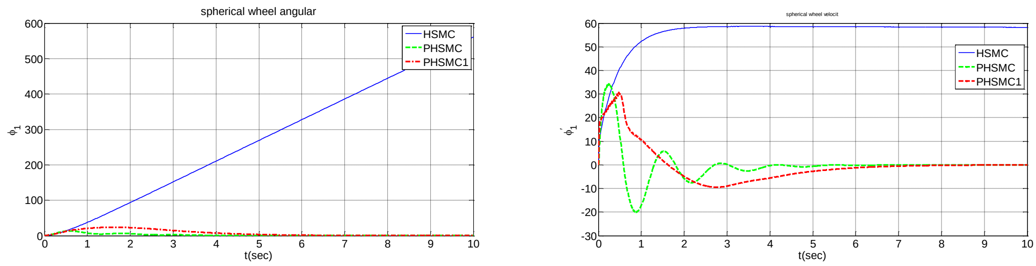

Figure 1-2 Angular position and velocity of the spherical wheel of the first dimension

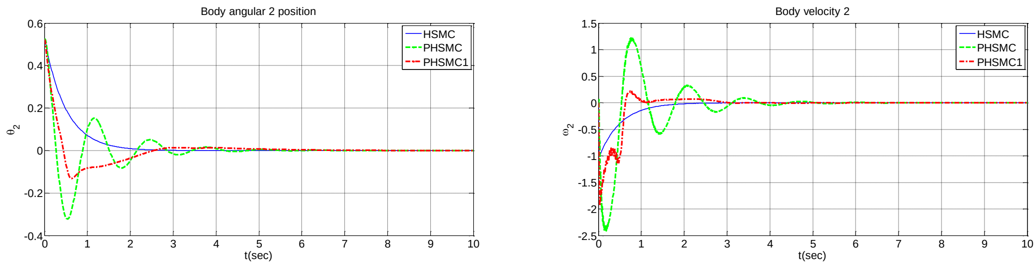

Figure 1-3 Angular position and velocity of the body of the second dimension

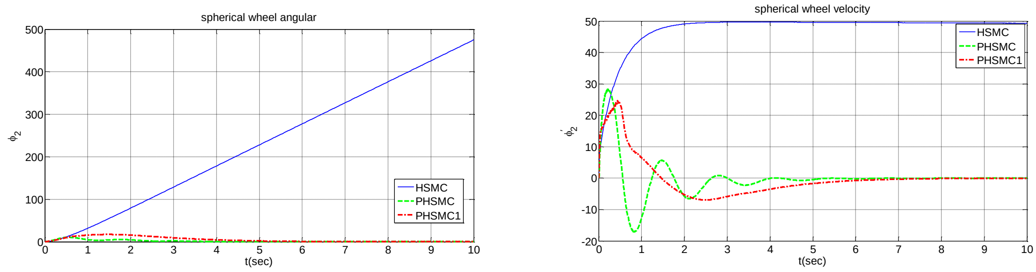

Figure 1-4 Angular position and velocity of the spherical wheel of the second dimension

The formulations of CSMC, PCSMC, and PCSMC1 are omitted in this report, due to the similarity with those of HSMC, PHSMC, and PHSMC1. For comparison of CSMC, PCSMC, and PCSMC1, simulations have been carried out with the period $2T = 1/500\,\mathrm{sec}$. Figures 2-1 and 2-3 show that the angular position and velocity of the body of both dimensions converge to zero for all controls. We also know that the convergent rate is fastest for the CSMC as expected. Undesired constant speed of the spherical wheel for the CSMC is also existed in Figures 2-2 and 2-4. However, the position control of the under-actuated spherical robot can also be implemented by the proposed PCSMC and PCSMC1.

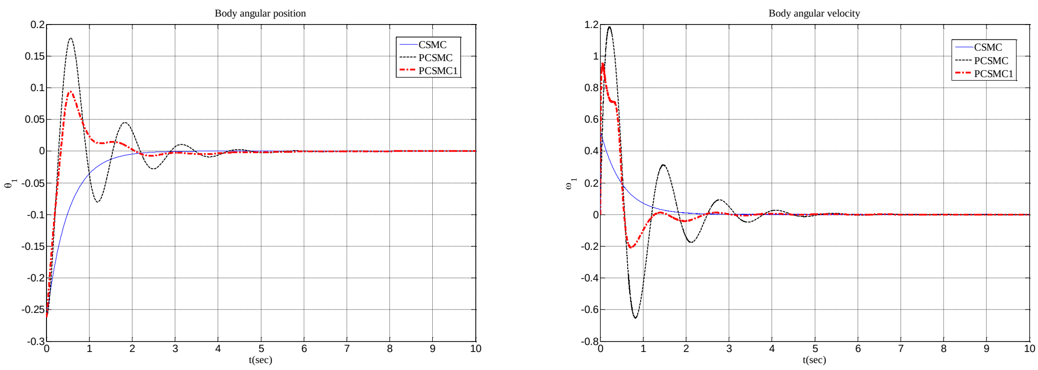

Figure 2-1 Angular position and velocity of the body of the first dimension

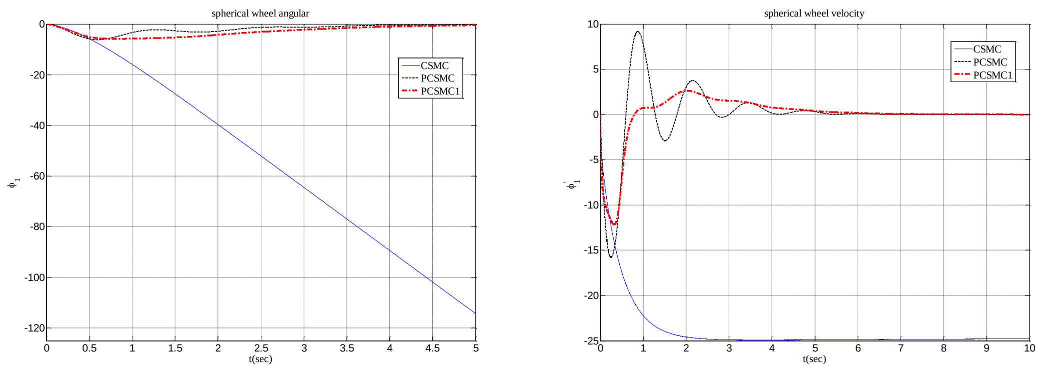

Figure 2-2 Angular position and velocity of the spherical wheel of the first dimension

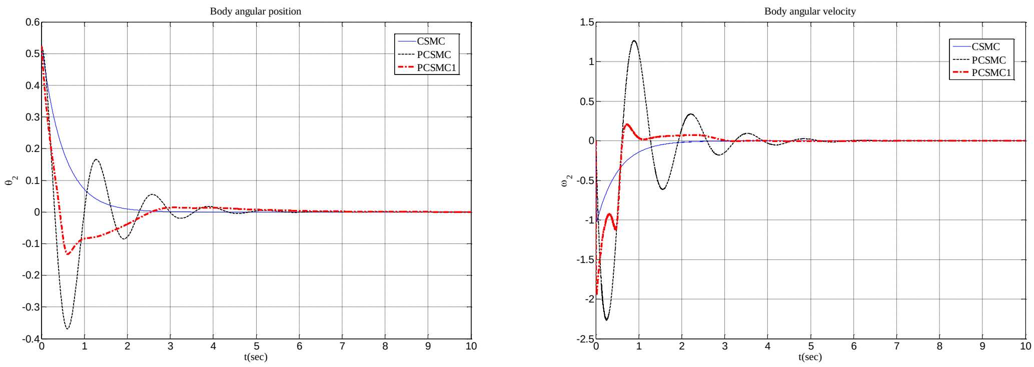

Figure 2-3 Angular position and velocity of the body of the second dimension

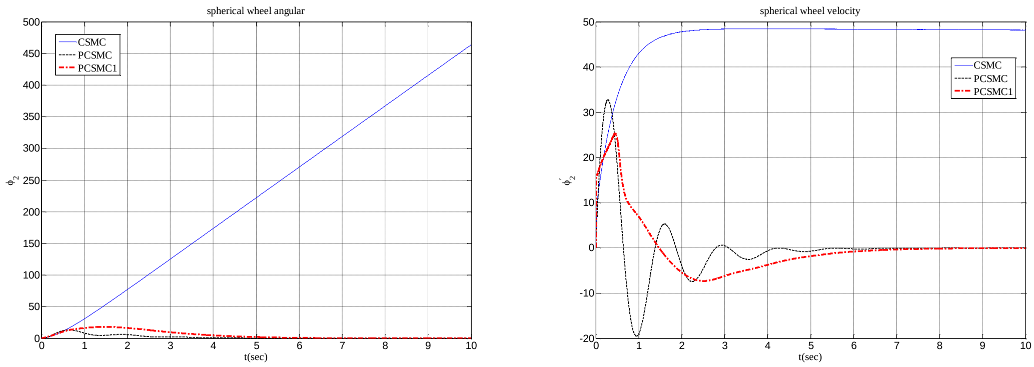

Figure 2-4 Angular position and velocity of the spherical wheel of the second dimension

[1] Chia-Wen Wu, Zhong-Wei Qiu, Yen-Hsiang Wang, Po-Hsiang Hsu, and Chi-Kuang Hwang, “Modeling of a spherical robot driven by Omni wheels”, International Conference on Machine Learning and Cybernetics (ICMLC), Volume 3, pp. 1256 – 1260, 2011.

[2] Chi-Hua Wang, Yu-Hsiang Lin, Kun-Shu Huang, Bore-Kuen Lee, Kuo-Bin Lin, and Chi-Kuang Hwang, “Constant speed VSC of a spherical robot driven by Omni wheels”, International Conference on Machine Learning and Cybernetics (ICMLC), Volume 3, pp. 1214 – 1219, 2011.

國科會補助計畫衍生研發成果推廣資料表

日期:2011/10/29

國科會補助計畫	計畫名稱: 利用全向輪推動之球輪機器人機構與控制設計	
	計畫主持人: 黃啟光	
	計畫編號: 99-2221-E-216-008-	學門領域: 智慧型機器人

無研發成果推廣資料

99 年度專題研究計畫研究成果彙整表

計畫主持人：黃啟光	計畫編號：99-2221-E-216-008-
計畫名稱：利用全向輪推動之球輪機器人機構與控制設計	

成果項目			量化：實際已達成數（被接受或已發表）	量化：預期總達成數(含實際已達成數)	量化：本計畫實際貢獻百分比	單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	1	1	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	1	1	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	2	2	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	2010 台北國際發明展自行車輔助系統銀牌

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標（請說明，以 100 字為限） 　　□實驗失敗 　　□因故實驗中斷 　　□其他原因 說明：
2. 研究成果在學術期刊發表或申請專利等情形： 論文：■已發表 □未發表之文稿 □撰寫中 □無 專利：□已獲得 ■申請中 □無 技轉：■已技轉 □洽談中 □無 其他：（以 100 字為限） [1] ' Modeling of a spherical robot driven by Omni wheels' , International Conference on Machine Learning and Cybernetics (ICMLC), Volume 3, pp. 1256 - 1260, 2011. [2] ' Constant speed VSC of a spherical robot driven by Omni wheels' , International Conference on Machine Learning and Cybernetics (ICMLC), Volume 3, pp. 1214 - 1219, 2011.
3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限） 目前正在進行分層滑模控制及串聯滑模控制，然而這兩種滑模控制容易造成主體姿態部分收斂太快為直立，進而造成球的定速問題，而非所需求的定位控制。本計畫修改這兩種的狀態切換法則為週期性的，就可以克服上述的問題。換言之，球輪的位置控制與主體姿態控制，在週期放鬆特性下可享有非嚴格收斂的機會。尤其是放鬆主體姿態的嚴格收斂速度，可大大避免球的定速問題，進而達成位置控制。