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一個計算隨機幾何圖上子圖出現機率的新方法及其極限(I)
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行政院國家科學委員會補助專題研究計畫 √ 成 果 報 告
□期中進度報告

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成果報告類型(依經費核定清單規定繳交)：√精簡報告　□完整報告

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摘要

隨機幾何圖上有一個基本和重要的研究問題是：分析隨機幾何圖上子圖出現的次數。例如，無線電網路 IEEE 802.11 CSMA/CA 通訊協定有兩個有名的問題： 隱藏節點問題和暴露節點問題。這兩種問題都和精確估算隨機幾何圖上子圖出現的次數有關。本計劃首次有系統地討論如何估算隨機幾何圖上某一特殊子圖出現次數的方法。首次嘗試精確地估算隨機幾何圖上子圖出現的次數。將估算隨機幾何圖上子圖出現次數的成果，運用於無線網路的分散式圖論演算法的設計上。同時討論邊界效應問題。

Abstract

This project undergoes quantitative analyses on fundamental properties of ad hoc networks including estimating the number of hidden-terminal pairs, the number of exposed-terminal sets, the number of neighboring nodes supporting triangle routes, and the extents of coverage and connectivity. To obtain these results, we propose a paradigm to systematically derive exact formulas for a great deal of subgraph probabilities of random geometric graphs. In contrast to previous work, which established asymptotic bounds or approximation, we obtain closed-form formulas that are fairly accurate and of practical value.

Keywords: Ad hoc networks, sensor networks, analytical method, random geometric graphs, connectivity, coverage, performance evaluation, hidden terminal, exposed terminal, quantitative analysis

1. Introduction

A *geometric graph* $G=(V, r)$ consists of nodes placed in 2-dimension space R^2 and edge set $E=\{(i, j) \mid d(i, j)\le r$, where $i, j\in V$ and $d(i, j)$ denotes the Euclidian distance between node i and node $j\}$. Let $X_n=\{x_1, x_2, \ldots, x_n\}$ be a set of independently and uniformly distributed random points. We use $\Psi(X_n, r, A)$ to denote the *random geometric graph* (RGG) [29] of n nodes on X_n with radius r and placed in an area A. RGGs consider geometric graphs on random point configurations. Applications of RGGs include communications networks, classification, spatial statistics, epidemiology, astrophysics, and neural networks [29].

A RGG $\Psi(X_n, r, A)$ is suitable to model an ad hoc network $N=(n, r, A)$ consisting of n mobile devices with transmission radius r unit length that are independently and uniformly distributed at random in an area A. When each vertex in $\Psi(X_n, r, A)$ represents a mobile device, each edge connecting two vertices represents a possible communication link as they are within the transmission range of each other. A random geometric graph and its representing network are shown in Figure 1. In the example, area A is a rectangle that is used to model the deployed area such as a meeting room. Area A, however, can be a circle, or any other shape, and even infinite space.

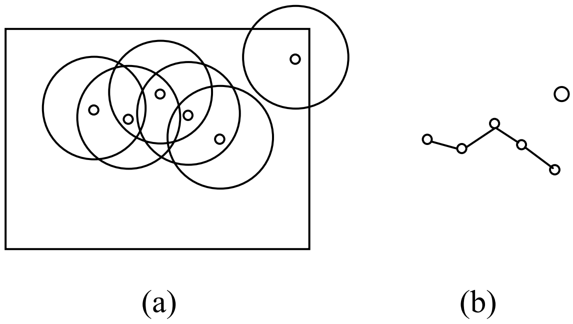

Figure 1. (a) An ad hoc network N=(6, r, A), where A is a rectangle. (b) Its associated random geometric graph $\Psi(X_6, r, A)$.

Many fundamental properties of ad hoc networks are related to subgraphs in RGGs. For example, the IEEE 802.11 CSMA/CA protocol suffers from the hidden and the exposed terminal problem [41, 45]. The hidden terminal problem is caused by concurrent transmissions of two nodes that cannot sense each other but transmit to the same destination. We call such two terminals a *hidden-terminal pair*. The existence of hidden-terminal pairs in an environment seriously results in garbled messages and increases communication delay, thus degrading system performance [24, 25, 45].

Quantitative analyses on specific subgraphs of a given RGG are of importance for understanding and evaluating the fundamental properties of MANETs. There is extensive literature on the subgraph probability of RGGs [29]. Penrose had shown that, for arbitrary feasible connected Γ with k vertices, the number of induced subgraphs isomorphic to Γ satisfies a Poisson limit theorem and a normal limit theorem [29]. To the best of our knowledge, previous related results are all asymptotic or approximate.

In the project, we make the first attempt to propose a paradigm to systematically derive the exact formulas for a great deal of subgraph probabilities in RGGs. In contrast to previous asymptotic bounds or approximation, the closed-form formulas we derived are fairly accurate and of practical value. With the paradigm, we undergoes quantitative analyses on fundamental properties of ad hoc networks including the number of hidden-terminal pairs, the number of exposed-terminal sets, the number of neighboring nodes supporting triangle routes, and the extents of coverage and connectivity.

Computing the probability of occurrence of RGG subgraphs is complicated by the assumption of finite plane. For example, one device in Figure 1 is deployed nearby the boundary of rectangle A so that its radio communication range (often modeled by a circle) is not properly contained in A. This is due to *border effects*, which complicate the derivation of closed formulas; therefore, previous discussions usually circumvent the border effects by using *torus convection* [1, 20]. Torus convention models the network topology in a way that nodes nearby the border are considered as being close to nodes at the opposite border and they are allowed to establish links. Most of the time, we adopt *torus convention* to deal with border effects in the report. However, we also obtain an exact formula for the single edge probability of RGGs when confronting the border effects.

The rest of this report is organized as follows. In Section 2, some definitions and notations are introduced. In Section 3, we briefly survey related results on RGGs. A method for computing the subgraph probability of RGGs with torus convention is presented in Section 4. Section 5 presents those derivations when confronting border effects. Finally, Section 6 concludes the report.

2. Definitions and notations

A *graph* $G=(V, E)$ consists of a finite nonempty vertex set V and edge set E of unordered pairs of distinct vertices of V. A graph $G=(V, E)$ is labeled when the $|V|$ vertices are distinguished from one another by names such as $v_1, v_2, \ldots, v_{|V|}$. Two labeled graphs $G=(V_G, E_G)$ and $H=(V_H, E_H)$ are identical, denoted by $G=H$ if $V_G= V_H$ and $E_G=E_H$. A graph $H=(V_H, E_H)$ is a *subgraph* of $G=(V_G, E_G)$ if $V_H\subseteq V_G$ and $E_H\subseteq E_G$. Suppose that V' is a nonempty subset of V. The subgraph of $G=(V, E)$ whose vertex set is V' and whose edge set is the set of those edges of G that have both ends in V' is called the subgraph of G *induced* by V', denoted by $G_{V'}$. The *size* of any set S is denoted by $|S|$. The *degree* of a vertex v in graph G is the number of edge incident with v. The notation $\binom{n}{m}$ denotes the number of ways to select m from n distinct objects.

The *subgraph probability* of RGGs is defined as follows. Let $\Omega=\{G_1, G_2, \ldots, G_k\}$ represent every possible labeled graphs of $\Psi(X_n, r, A)$, where $k=2^{\binom{n}{2}}$. When G_x is a labeled subgraph in Ω, we use $\Pr(G_x)$ to denote the probability of the occurrence of G_x. Suppose $S\subseteq V$ and $T\subseteq V$, we define $\Pr(G_s)=\sum_{\forall G_w\in\Omega \text{ and } G_s=G_t\subseteq G_w}\Pr(G_w)$, when $1\leq w\leq k$.

3. Related work in RGG

To the best of our knowledge, previous results on RGGs are all asymptotic and approximate except [49, 50]. We summary related results as follows.

A book written by Penrose [29] provides and explains the theory of random geometric graphs. Graph problems considered in the book include subgraph and component counts, vertex degrees, cliques and colorings, minimum degree, the largest component, partitioning problems, and connectivity and the number of components.

For n points uniformly randomly distributed on a unit cube in $d\geq 2$ dimensions, Penrose [32] showed that the resulting geometric random graph G is k-connected and G has minimum degree k at the same time when $n\rightarrow\infty$. In [9, 10], Díaz *et al.* discussed many layout problems including minimum linear arrangement, cutwidth, sum cut, vertex separation, edge bisection, and vertex bisection in random geometric graphs. In [11], Díaz *et al.* considered the clique or chromatic number of random geometric graphs and their connectivity.

Some results of RGGs can be applied to the connectivity problem of ad hoc networks. In [39], Santi and Blough discussed the connectivity problem of random geometric graphs $\Psi(X_n, r, A)$, where A is a d-dimensional region with the same length size. In [1], Bettstetter investigated two fundamental characteristics of wireless networks: its minimum node degree and its k-connectivity. In [12], Dousse *et al.* obtained analytical expressions of the probability of connectivity in the one dimension case. In [18], Gupta and Kumar have shown that if $r=\sqrt{\frac{\log n + c(n)}{\pi n}}$, then the resulting network is connected with high probability if and only if $c(n)\to\infty$. In [47], Xue and Kumar have shown that each node should be connected to $\Theta(\log n)$ nearest neighbors in order that the overall network is connected.

Recently, Yen and Yu have analyzed link probability, expected node degree, and expected coverage of MANETs [49]. In [48], Yang has obtained the limits of the number of subgraphs of a specified type which appear in a random graph.

4. Computing subgraph probability

In the section, we develop a paradigm for computing subgraph probability of RGGS. First of all, we are to prove that the occurrences of arbitrary two distinct edges in RGGs are independent in the next subsection. The property of edge independence greatly simplifies our further calculations. For simplicity, we always assume that A is sufficiently large to properly contain a circle with radius r in a $\Psi(X_n, r, A)$ throughout the report; that implies $\pi r^2 \leq |A|$. In the report, notation E_i (E_i') denotes the event of the occurrence (absence) of edge e_i.

Since we adopt torus convention to avoid border effects in the section, single-edge probability in RGG is obtained trivially and listed below.

Theorem 1: We have $\Pr(E_j)=\pi r^2/|A|$, for an arbitrary edge $e_j=(u, v)$ and $u\neq v$, in a $\Psi(X_n, r, A)$.

4.1 Edge independence in RGGs

The following theorem shows that the occurrences of arbitrary two distinct edges in RGGs are independent even if they share one end vertex.

Theorem 2: For arbitrary two distinct edges $e_i=(u, v)$ and $e_j=(w, x)$ in a $\Psi(X_n, r, A)$, we have $\Pr(E_iE_j)=\Pr(E_i)\Pr(E_j)$.

Theorem 2 indicates that the occurrences of arbitrary two distinct edges in RGGs are independent. The result is somewhat difficult to be accepted as facts at first glance for some scholars. For example, Santi and Blough [39] claimed that the occurrences of two distinct edges $e_1=(u, v)$ and $e_2=(u, w)$ are correlated by observing that

if

S_1: ($d(u, v)<d(u, w)$)

then

S_2: (the existence of e_2 (E_2) implies the existence of e_1 (E_1)).

In logical terms, the statement (if S_1 then S_2) is true under all its interpretations (that is, the statement is a tautology) [6]; however, that does not necessarily imply the truth of S_2 and the conclusion that any two distinct edges are dependent.

The falsity of their deduction can be proved by contradiction. Given four distinct nodes u, v, w, and x, the statement

"If

S_3: ($d(u, v)<d(w, x)$) then

S_4: (the existence of $e_1=(w, x)$ implies the existence of $e_2=(u, v)$)."

is a tautology. Provided that Santi and Blough's deduction is correct, we conclude that S_4 is true.

Consequently, the false statement "two independent edges (*i.e.* (w, x) and (u, v)) are dependent" were true. A contradiction occurs.

Note that Theorem 2 does not imply that the occurrences of more than two edges in RGGs are also independent. In fact, we will show their dependence later.

By Theorem 1 and 2, we obtain the probability of two-edge subgraphs immediately.

Corollary 3: For arbitrary two distinct edges $e_i=(u, v)$ and $e_j=(w, x)$ in a $\Psi(X_n, r, A)$, we have $\Pr(E_iE_j)= (\pi r^2/|A|)^2$.

4.2 Base subgraphs

In this subsection, we consider eight labeled subgraphs with three vertices as *base subgraphs*, the probabilities of which will be used to compute the probability of larger subgraphs later. Based on the number of edges included, subgraphs of three vertices can be classified into four groups: a triangle (c_3), an induced path of length two (p_2), an edge with an isolated vertex (p_1+I_1), and three isolated vertices (I_3) (See Figure 2).

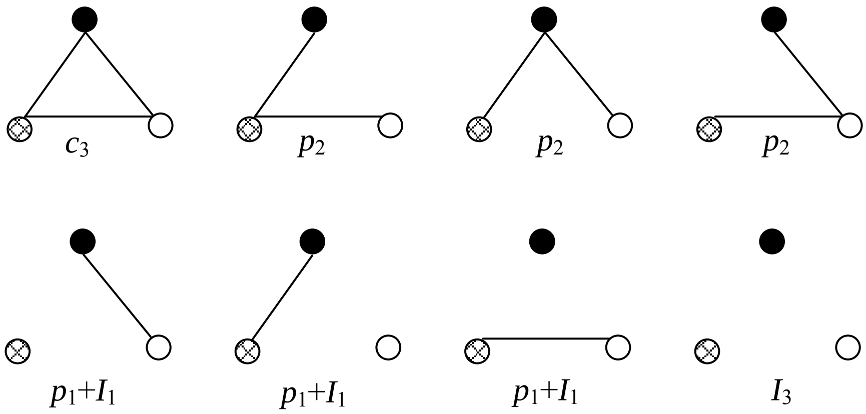

Figure 2. Eight base subgraphs.

To compute the probability of c_3, we need the following lemma. If one of two equal-sized circles in the place contains the center of the other, we call them two *properly intersecting circles*.

Lemma 4 [50]: The expected overlapped area of two properly intersecting circles with the same radius r is $\left(\pi - \frac{3\sqrt{3}}{4}\right)r^2$ in a $\Psi(X_n, r, A)$.

The following conditional probability is a consequence of Lemma 4.

Lemma 5: For three distinct edges $e_i=(u, v)$, $e_j=(u, w)$, and $e_k=(v, w)$ in a $\Psi(X_n, r, A)$, we have $\Pr(E_iE_j | E_k)=\left(\pi - \frac{3\sqrt{3}}{4}\right)r^2/|A|$, where $u\neq v\neq w$.

The probability of the first base subgraph c_3 (triangle) can then be obtained.

Theorem 6: For three distinct edges $e_i=(u, v)$, $e_j=(u, w)$, and $e_k=(v, w)$ in a $\Psi(X_n, r, A)$, we have $\Pr(E_iE_jE_k)=\left(\pi - \frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2$, where $u\neq v\neq w$.

Next, we consider the subgraph of an edge with an isolated vertex (p_1+I_1).

Theorem 7: For three distinct edges $e_i=(u, v)$, $e_j=(u, w)$, and $e_k=(v, w)$ in a $\Psi(X_n, r, A)$, we have $\mathrm{P}(E_iE_j'E_k')=\frac{\pi r^2}{|A|}(1-\frac{\pi r^2}{|A|}-\frac{3\sqrt{3}}{4|A|}r^2)$, where $u\neq v\neq w$.

We have shown that the occurrences of two distinct edges in a $\Psi(X_n, r, A)$ are independent (Theorem 2). The next theorem, however, shows that edge independence does not exist for subgraphs with three or more edges.

Theorem 8: The occurrences of arbitrary three distinct edges in a $\Psi(X_n, r, A)$ are dependent.

The next base subgraph we considered is an induced path p_2, which will be used to model a hidden-terminal pair.

Theorem 9: For arbitrary three distinct edges $e_i=(u, v)$, $e_j=(u, w)$, and $e_k=(v, w)$ in a $\Psi(X_n, r, A)$, we have $\Pr(E_iE_jE_k')=\left(\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2$, where $u\neq v\neq w$.

The last base subgraph we considered is I_3.

Theorem 10: For arbitrary three distinct edges $e_i=(u, v)$, $e_j=(u, w)$, and $e_k=(v, w)$ in a $\Psi(X_n, r, A)$, we have $\Pr(E_i'E_j'E_k')=1-\frac{\pi r^4}{|A|}-\frac{\frac{3\sqrt{3}}{4}}{|A|^2}\pi r^4$, where $u\neq v\neq w$.

5. Computing subgraph probability in the face of border effects

In the section, we restrict the deployed area A to an $l\times m$ rectangle. We make an attempt to face border effects and obtain a closed-form formula of computing the single edge probability of RGGs. The results derived in the section will be used to measure the extent of coverage and connectivity of ad hoc networks later.

For clarity, the main result and its corollaries are listed before their proofs.

Theorem 11: Given a $\Psi(X_n, r, A)$ and an $l\times m$ rectangle A, the single edge probability considering border effects is $\frac{\frac{1}{2}r^4-\frac{4}{3}lr^3-\frac{4}{3}mr^3+\pi r^2ml}{m^2l^2}$.

Corollary 12: The average (expected) degree of a vertex in a $\Psi(X_n, r, A)$ considering border effects is $(n-1)\times(\frac{\frac{1}{2}r^4-\frac{4}{3}lr^3-\frac{4}{3}mr^3+\pi r^2ml}{m^2l^2})$, where A is an $l\times m$ rectangle.

Corollary 13: The expected edge number of a $\Psi(X_n, r, A)$ considering border effects is $(\frac{n(n-1)}{2})\times(\frac{\frac{1}{2}r^4-\frac{4}{3}lr^3-\frac{4}{3}mr^3+\pi r^2ml}{m^2l^2})$, where A is an $l\times m$ rectangle.

To obtain these results, we first derive some necessary lemmas. Let $X_n=\{x_1, x_2, \ldots, x_n\}$ be a set of independently and uniformly distributed random points in a given $\Psi(X_n, r, A)$, where $x_i=(X_i, Y_i)$ and $0\leq X_i\leq l$ and $0\leq Y_i\leq m$, for $1\leq i\leq n$. Clearly, X_i's (and Y_i's) are independent, identically distributed random variables with probability density function (p.d.f.) $f(x)=1/l$ ($g(y)=1/m$) over the range $[0, l]$ ($[0, m]$).

Lemma 14: Given a $\Psi(X_n, r, A)$ and any two distinct nodes $x_i=(X_i, Y_i)$ and $x_j=(X_j, Y_j)$, we have

Pr[$|X_i$-$X_j|\le z$]=$\frac{-z^2+2lz}{l^2}$ and Pr[$|Y_i$-$Y_j|\le w$]=$\frac{-w^2+2mw}{m^2}$ where $0\le z\le l$ and $0\le w\le m$.

Lemma 16: Given a $\Psi(X_n, r, A)$ and any two distinct nodes $x_i=(X_i, Y_i)$ and $x_j=(X_j, Y_j)$, we have that: (1) the p.d.f. of $(X_i\text{-}X_j)^2$ is $f(u)=\frac{lu^{-\frac{1}{2}}-1}{l^2}$ where $0\le u\le l^2$, and (2) the p.d.f. of $(Y_i\text{-}Y_j)^2$ is $g(v)=\frac{mv^{-\frac{1}{2}}-1}{m^2}$, where $0\le v\le m^2$.

Lemma 15 [43]: $\int u^{-\frac{1}{2}}\sqrt{a^2-u}du=u^{\frac{1}{2}}\sqrt{a^2-u}+a^2\sin^{-1}\frac{\sqrt{u}}{a}+c$, where c is a constant.

Finally, we prove the main theorem of the section as follows.

Theorem 12: Given a $\Psi(X_n, r, A)$ and an $l\times m$ rectangle A, the single edge probability is $\frac{\frac{1}{2}r^4-\frac{4}{3}lr^3-\frac{4}{3}mr^3+\pi r^2ml}{m^2l^2}$, when border effects are considered.

We conclude that border effect does affect the value of the single edge probability of $\Psi(X_n, r, A)$. If A is an $l\times m$ rectangle, the difference between the single edge probabilities with and without avoiding border effects (by adopting torus convention) is $\frac{\frac{4}{3}mr^3+\frac{4}{3}lr^3-\frac{1}{2}r^4}{m^2l^2}$.

6. Conclusions

We have proposed a paradigm for computing the subgraph probabilities of RGGs, and have shown its applications in finding fundamental properties of wireless networks. We are surprised at finding some interesting properties:

1. The occurrences of two distinct edges in RGG are independent.
2. The occurrences of three or more distinct edges in RGG are dependent.
3. Probabilities of some specific subgraphs in RGG can be estimated accurately.

Many interesting subgraph probabilities and their applications in MANETs are still uncovered. For example, we are now interested in accurately estimating the diameter of RGGs. We also believe that the techniques developed in the report can be exploited to conduct quantitative analysis on other fundamental properties of wireless ad hoc networks.

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行政院國家科學委員會補助國內專家學者出席國際學術會議報告

2007 年 4 月 30 日

附件三

報告人姓名	俞征武	服務機構及職稱	中華大學資訊工程系 副教授
會議 時間 地點	4/7~4/9 China, Hangzhou	本會核定補助文號	中華研國字第 75 號
會議名稱	(中文) 2007 年國際組合演算法機率與實驗方法會議 (英文) IntErnational Symposium on Combinatorics, Algorithms, Probabilistic and Experimental Methodologies (ESCAPE 2007)		
發表論文題目	(中文)一個的多步無線網路新的量化分析方法 (英文) Quantitative Analysis of Multi-Hop Wireless Networks Using a Novel Paradigm		

報告內容應包括下列各項:

一、參加會議經過

2007 年國際組合演算法機率與實驗方法會議於 4 月 7 日到 4 月 9 日在 China, Hangzhou 舉行。4 月 7 日我前往 Hangzhou 浙江大學將行裏放置於 Lingfeng Hotel 並前往玉泉校區的數學系館。本次會議我有兩篇論文發表，第一篇於 4 月 7 日下午 15:40-17:00。與 session chair Prof. Dr. Winfried Hochstattler (chair of discrete mathematics and optimization, FernUniversitat in Hagen) 寒暄後，逕行發表論文。

4 月 8 日晨我前往 Dept. of Information and Electronics Engineering 將一份同事之禮物送給陳抗生教授。今日雖爲星期天但陳抗生教授也到校從事研究,與陳教授促膝而談，我深深感受到他治學的胸襟及精神。這也是我此趟參加 ESCAPE 最大的收穫。中午值 Theoretical Computer Science 的 Mike Paterson 及張真誠(IEEE fellow)及詹昭文教授並與之共進午餐。向張教授請益相當多。也與 Xidian University 之 Huaxi Gu 有相當多的互動。下午大會安排遊西湖，與 Prof. 楊波艇 (Boting Yang Department of computer science, University of Regina, Canada)有相當大的互動，他原爲數學系目前研究 treewidth。晚宴於山外山並頒發 the best paper award。

4 月 9 日晨我的第二篇論文發表,與優秀的浙江大學學生有良好的互動.中午向 chair Prof. Guochuan Zhang 告別後直接到 Xiao-Shan 機場經澳門飛回台灣。

二、與會心得

中國近年對舉辦國際 CONFERENCE 相當積極。台灣學術界要加油了。

三、考察參觀活動(無是項活動者省略)

4/8 與會學者相邀前往西湖著名風景。

四、建議

台灣學術界應多參與國際中重要會議，甚至以大學爲單位，有利於展現學術力量。

五、攜回資料名稱及內容

六、其他