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一個計算隨機幾何圖上子圖出現機率的新方法及其極限(II)

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摘要

隨機幾何圖的應用包含通訊網路(communication networks)、隨意網路(ad hoc networks)、感測網路(sensor networks)、分類技巧(classification)、空間統計(spatial statistics)、流行病學(epidemiology)、 天文物理學(astrophysics)、和類神經網路(neural networks)等。隨機幾何圖可用於表示無線電網路系統。隨機幾何圖上有一個基本和重要的研究問題是：分析隨機幾何圖上子圖出現的機率。例如，無線電網路 IEEE 802.11 CSMA/CA 通訊協定有兩個有名的問題： 隱藏節點問題(the hidden terminal problem)和暴露節點問題(the exposed terminal problem)。這兩種問題都和隨機幾何圖上某一特殊子圖出現的次數有關。而利用隨機幾何圖上子圖出現的機率有助於計算子圖出現的次數。本計畫分兩年執行。前一年專注於「發展精確估算隨機幾何圖上子圖出現的機率及次數的新方法及探討此方法的極限」第二年(本年)將專注於「解決隨機幾何圖和無線網路上有密切關係的圖論問題及設計其相關演算法」。應用第一年計算隨機幾何圖上出現機率及次數的方法，來分析和網路有關的特殊圖論性質。

Abstract

Random geometric graphs (RGG) contain vertices whose points are uniformly distributed in a given plane and an edge between two distinct nodes exists when their distance is less than a given positive value. RGGs are appropriate for modeling multi-hop wireless networks consisting of *n* mobile devices with transmission radius *r* unit length that are independently and uniformly distributed randomly in an area. This work presents a novel paradigm to compute the subgraph probability in RGGs. In contrast to previous asymptotic bounds or approximation, the closed-form formulas we derived herein are fairly accurate and of practical value. In this report, we aim to develop a paradigm which can be used to make quantitative analyzes on the fundamental properties of multi-hop wireless networks.

Keywords: Random geometric graphs, subgraph counting, subgraph probability

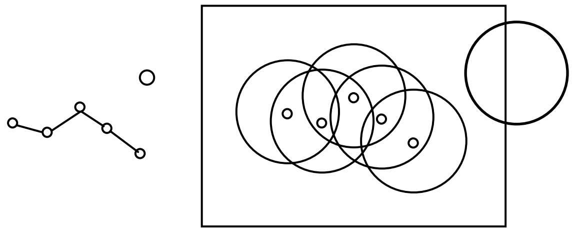
1. 前言

最近幾年來，一個全新的圖學模型：*隨機幾何圖(random geometric graphs)*[22] 被學者先後提出來。隨機幾何圖有相當多的不同的定義，我們採用以下的定義[22]：

1. 有n個點(node) uniformly randomly分佈於已知的幾何空間中。令X_0, X_1, …, X_{n-1} 為此n點{0, 1, 2, …, n-1}的座標，則皆可視為獨立的隨機變數(independent random variable)。注意此幾何空間有相當多的可能性；有學者定義為為方形，矩形，圓形，一維線段，或高於二維，或無邊界，或一個封閉空間。
2. 一個隨機幾何圖$G=(V, E)$，其中V是由上面n個點={0, 1, 2, …, n-1}所組成，而如果任V中的兩個點(node) i和j的距離小於r則(i, j)之間存在一個edge。也就是說如果$d(X_i, X_j)\leq r$則$(i, j)\in E$，此處 d函數代表兩個node之間的歐基里德距離(Euclidean distance)。

隨機幾何圖的應用相當廣，包含通訊網路(communication networks)、隨意網路(ad hoc networks)、感測網路(sensor networks)、分類技巧(classification)、空間統計(spatial statistics)、流行病學(epidemiology)、 天文物理學(astrophysics)、和類神經網路(neural networks)等[22]。

隨機幾何圖可用於表示無線電網路系統。例如將一個無線網路佈置在一個矩型的活動會場上，此會場可視爲出現在前面定義中的一個幾何上的空間。每個點視爲一個無線電收發器(transceiver)，而其收發功率範圍(power range)皆爲 *r*。任兩點有一條線(edge)代表此兩個收發器可相互直接通訊。整個會場一共有 *n* 個收發器隨機 uniformly 地分配於活動會場上。簡言之，此一個無線網路系統 $N=(n, r, l, m)$代表有 *n* 個傳輸半徑爲 *r* 的無線電收發器，此設備是獨立且 uniformly 分配在一個長爲 *l* 寬爲 *m* 的空間中。一個隨機幾何圖合適於代表一個無線電網路其隨機部署(如感測網路)或隨意移動(如隨意網路)的動態本質。在圖一中，左圖代表右邊無線網路系統的一個隨機幾何圖。



圖一、隨機幾何圖及其代表的無線網路系統。

無線電網路的重要性和普及性日益劇增，因此隨機幾何圖相關的理論研究問題也受到廣泛的重視。一般而言，無線電網路上的基本問題，常可對應到某一個圖論問題。例如，如果我們問當功率範圍(power range) *r* 需要調到多大以上，則隨機部署的無線網路是否完全連接？若用圖學的術語就是問：當 *r* 值多大之時，此隨機幾何圖是強連接的(strongly connected)的機率會極高。此問題稱爲*隨機幾何圖的連接問題*[29]。此連接問題在無線網路(mobile ad hoc or sensor networks)上有相當多的應用，包含：省電機制設計(power saving)、網路佈置(deployment)、網路產值(throughput)之估算、及網路路由(routing)等問題[7, 10, 13, 14, 17, 24, 25, 26, 29]。

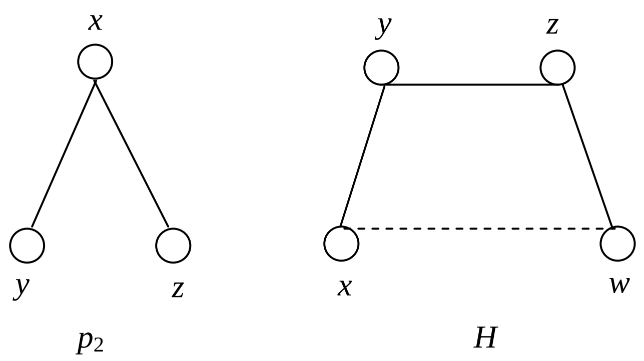
2. 研究目的

隨機幾何圖上有一個基本問題：**計算隨機幾何圖上子圖出現的機率**。我們注意到，若能準確地估算隨機幾何圖上子圖出現的機率，無線電網路的許多基本問題，將會有一個統一的且有效的量化分析平台。例如，無線電網路 IEEE 802.11 CSMA/CA 通訊協定存在兩個有名的問題： 隱藏節點問題(the hidden terminal problem) [37-39]和暴露節點問題(the exposed terminal problem) [36]。這兩種問題都和隨機幾何圖上子圖出現的次數有關。

隱藏節點問題的發生，是因爲兩個節點不能感測對方的存在，而卻同時傳送給同一個接收器的問題。我們稱此兩節點爲*隱藏節點配對*(a *hidden-terminal pair*)。隱藏節點配對的存在嚴重造成無線網路的資訊毀損(garbled messages)及通訊延遲(communication delay)，因而導致整個系統的效能(system performance)下降 [36-39]。暴露節點問題因爲兩個節點可以相互感測對方，卻被不當禁止傳送給不同且不會相戶干擾的接收節點[36]。暴露節點問題導致網路頻寬，受到不必要的限制。我們稱這些相關的節點爲*暴露節點集合*(*the exposed-terminal set*).

當我們利用一個圖 $G=(V, E)$代表一個無線網路時，隱藏節點配對和暴露節點集合都可以利用一個子圖 (subgraph) 來表示(圖二)。首先，一個隱藏節點配對可以利用一對 edges (x, y)和 (x, z)來表示。其中$(x, y)\in E$ 和$(x, z)\in E$，但是$(y,$

$z) \notin E$。從圖論的名詞上可以說是此 induced subgraph 為一個長度為 2 的路徑(path)(圖二左圖)。相同地，一個暴露節點集合也可以用一個四個 vertices 的子圖來代表。其中$\{x, y, z, w\} \subseteq V$ 而$\{(x, y), (y, z), (z, w)\} \subseteq E$,但是$(x, z) \notin E$ 而且$(y, w) \notin E$(圖二右圖)。



圖二、隱藏節點配對p_2和暴露節點集合H。

有少數學者發現：隱藏節點問題和暴露節點問題對無線網路效能的影響程度，和隱藏節點配對p_2和暴露節點集合H出現在網路上的次數有直接的關係[36, 37-39]。因此若我們可以準確地估算這些子圖出現的機率，將有助於正確評估無線網路效能及其它的基本性質。

可惜的是，前人對於估算隨機幾何圖上子圖出現次數的成果，都是非常粗略的估計值(asymptotic results)。如在[22]中第三章，Penrose認為任意子圖H其出現在隨機幾何圖上子圖出現次數滿足Poisson limit theorem(當$n^k r_n^{d(k-1)}$趨近一個常數時)和normal limit theorem(當$n^k r_n^{d(k-1)}$趨近無限大而r_n趨近 0 時，或當r_n是個常數時)。他們的粗略的估計值非常不精確，因此大大降低實際應用的時機及價值。較詳細的隨機幾何圖之相關研究，我們將在Appendix I中作一些摘要介紹[9-11, 29-32]。

在了解前人對隨機幾何圖所作的研究成果之後，我們有下列的看法：

1. 目前並無一套的方法，可用於經確估算隨機幾何圖子圖出現機率或次數。
2. 隨機幾何圖基本性質的研究仍不足。
3. 隨機幾何圖雖不是隨機圖，但兩者之間的共通特性和不同處，值得深入探索。
4. 先前的研究刻意忽視邊界效應的問題，因此並不符合實際的應用。
5. 如何運用隨機幾何圖於無線網路系統和應用上，目前人類所知有限。

若可對隨機幾何圖上一些特殊子圖出現的機率，作精準的估計。我們將有機會估計子圖出現的次數；進而有機會建立一個無線網路量化分析的理論。本計畫分兩年執行。第一年專注於「發展精確估算隨機幾何圖上子圖出現的機率及次數的新方法及探討此方法的極限」。第二年將專注於「解決隨機幾何圖和無線網路上有密切關係的圖論問題及設計其相關演算法」。

3. 文獻探討

有關隨機幾何圖的相關研究成果羅列如下：隨機幾何圖理論是一個正在起步中的學問。目前有一本 2003 年出版的專書，由 M.D. Penrose 所著作[22]。該書所討論的圖論問題包含：the longest nearest-neighbor link，the longest edge and total cost of the Euclidean Minimum Spanning Tree 等問題。其中 the longest edge of the Euclidean Minimum Spanning Tree 可用來解隨機幾何圖的連接問題，請參考該作者的相關著作[23]。

在 2003 年, Santi 和 Blough [29] 首先考慮一個在一維幾何上的隨機幾何圖(也就是 node 只分佈於一條直的線段上)何時會連結(connected)的問題。並討論何時此隨機幾何圖，可以大部分地連結(例如 90%是連成一個 connected component)。他們的公式是當 $rn>=l \ln l$ 時，整個圖連接的機率極高。但當 $rn<=(1-\varepsilon)l \ln l$ 則此圖不易連接,此地的ε為界於 1 到 0 的實數。他們也將此公式推廣於兩維和三維的問題上，只是這些公式都是假設 n 或 r 是無限大的(asymptoticlly)時推論而得；故當實際問題 n 或 r 是在一個普通大小的值時，他們的公式就值得檢驗是否仍然依舊正確可用。他們早期的研究也出現在此篇論文中[30]。

相當多文獻中，對隨機幾何圖的基本假設需要node分佈的狀況是Poisson分配，而不是先前定義的Uniform 分配。因此分佈點的數目是一個隨機變數，只有其期望值可估計。並無法實際掌握正確的數目，有時並不符合實際的使用狀況。如在 2002 年，Bettstetter [1]利用nearest neighbor method考慮隨機幾何圖上的minimum node degree，並將成果推廣到解*k*-connectivity 的問題；因為他利用Penrose在 1998 年發表的一個重要研究成果[23]：當隨機幾何圖成為*k*-connectivity時此圖的minimum node degree也同時大於等於*k*。注意此性質在隨機圖上也成立[2, 21]。在 2002 年，Dousse等人 [7] 同樣在一維空間上分析連結問題，但也須假設node分佈是一種Poisson分配。在 1998 年，Gupta 和Kumar當傳輸半徑設定為$((\log n+c(n))/n\pi^2)^{1/2}$ 時則整個網路連接的機率接近 1 若且為若$c(n)$趨近無限大[13]。在 1989 年，Philips等人也討論Poisson分配下，當網路需連結時node需要的平均個數[24]。同樣的問題在一維空間上被Piret討論過[25]。

除了討論隨機幾何圖的連結問題外，其他相關的研究雖並未直接用隨機幾何圖來討論，但本質上卻息息相關，常藏身於無線電網路的相關論文中[14, 17, 18, 19, 33]。如網路容量(network capacity)[14]問題也吸引到一些目光。當每個收發器的功率半徑變大時，則每個收發器的鄰居變多，而到目的收發器的距離也變近了，導致訊息需較少的時間來傳輸。但是相對的，在 MAC 層上對相鄰點的干擾也增加了，因此也使得每個 hop 傳輸時間增長。如何取得一個平衡點，而得到整個系統的產值(throughput)為最大是一個有趣的網路容量問題。在 1978 年, Kleinrock 等提出當 average degree 大約為 6 時,整個網路的產值為最大。因此他們稱 six 是一個神奇數字(a magic number) [17]。但 Philips[24]等認為並無 magic number 的存在，他們也計算出 average degree 大約為2.195~10.525 之間時，整個網路的產值為最大。Gupta 等也對此議題作深入的討論[14]。

在了解前人對隨機幾何圖所作的研究成果之後，我們有下列的看法：

6. 隨機幾何圖基本性質的研究仍不足。隨機幾何圖並不是學術界早已熟知的*隨機圖*[2, 8, 9, 21] (*random graphs*)是眾所週知。但若干文獻所提到出的理由，有模糊不清之處[29]，不為本計畫主持人所接受(請參考後文『Random graphs 及其不適用於無線網路的原因』該節所做的論述)。例如，隨機幾何圖中的任兩個 edge 出現的事件是相同的(uniform distribution)嗎？是獨

立(independent)出現的嗎？出現的機率有多大？文獻中的看法大多是否定的或闕如，但根據我們的實驗卻是指向另一個不同的答案。因此，我們認爲一些隨機幾何圖的基本性質並未被完整的討論過。

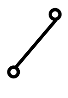
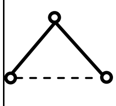
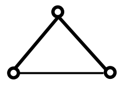
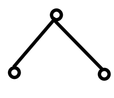
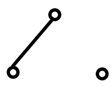
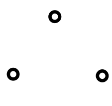
7. **隨機幾何圖雖不是隨機圖，但兩者之間的共通特性和不同處，值的深入探索**。例如，兩圖都有門檻函數(threshold function)[21]的特質及兩圖都有 k-connectivity 和 minimum degree 爲 k 同時出現的特質[2, 23]。比較兩圖的差異及參考隨機圖現在已被發現的性質，有助於快速發現及證明隨機幾何圖的一些基本特性。
8. **先前的研究刻意忽視邊界效應的問題**。一般學者在思考隨機幾何圖時，爲避免繁瑣的計算，都刻意不考慮邊界效應。但如此，將使得所有的估計不準確或落於不切實際的假設。
9. **如何運用隨機幾何圖於無線網路系統和應用上**。無線電網路系統上的問題常常可視爲圖論問題，例如圖色問題(the coloring problem)[3]常可用於無線電網路資源的最佳分配上。是故如何將隨機幾何圖善加運用於無線網路上，值得學者深入研究。

以下我們將對在本計畫中提及的隨機圖作較仔細的定義，並討論爲何隨機圖(Random graphs)並不適用於無線網路的原因。1959 Erdös and Rénye [8]引入了 random graphs 的理論之後便成一個新的數學領域. 甚至有一個期刊 Journal of Random Structures and Algorithms 是爲 random graphs 相關研究所成立的。一個常用來定義 random graphs 的方式爲 *The binomial random graph model* [21]: 給定 n 個不同的 vertex 和一個機率值 p，其值的範圍爲 $0<p<1$, 令整個樣本空間 Ω 包含所有 n 的 vertex 的所有可能出現的圖 G。如果 G 共有 q 個 edges，則 G 出現的機率爲 $P(G)$，定義如下： $P(G)=p^q(1-p)^{\binom{n}{2}-q}$.我們也可以用隨機過程(*random graph process*)來說明。一開始有 n 個不同的 vertex，但沒有任何 edge 出現。接下來，從未被加入的 edge 中，uniformly 隨機地選擇其一加入。一般認爲隨機幾何圖並不是 random graphs，因爲在 random graphs 中，任一 edge 出現的機率相同的，而且是獨立出現的。雖然隨機幾何圖上的 node 是獨立且 uniformly 隨機地出現的，但隨機幾何圖上的 edge 出現，一般認爲並不是機率相同的，而且是相依(dependent)出現的。

4. 研究方法

爲了解決無線網路上量化的分析問題，我們先利用第一年的成果加以擴充並用之於分析各種無線網路上的問題。首先將第一年的子圖機率列於下表。

Table 1. Probabilities of subgraphs with three vertices or less in a RGG.

Notation	p_1	E^2	c_3	p_2	E^1+I_1	I_3
G						
Pr(G)	$\pi r^2/\|A\|$	$(\pi r^2/\|A\|)^2$	$\left(\pi-\frac{3\sqrt{3}}{4}\right)\pi r^4/\|A\|^2$	$\left(\frac{3\sqrt{3}}{4}\right)\pi r^4/\|A\|^2$	$\frac{\pi r^2}{\|A\|}(1-\frac{\pi r^2}{\|A\|}-\frac{\frac{3\sqrt{3}}{4}}{\|A\|}r^2)$	$1-\frac{\pi r^4}{\|A\|}-\frac{\frac{3\sqrt{3}}{4}}{\|A\|^2}\pi r^4.$

我們發現可分析的子圖有一種特殊圖稱作Y-graph。我們將其定義及證明列於後。首先是Y-graph的定義。一個 class graph $G=(V, E_S, E_B)$ 是一個 *Y-graph* 如果此圖符合下列原則:

R1: All base subgraphs are Y-graphs.
R2: G_1+G_2 and G_1-G_2 are also Y-graphs if G_1 and G_2 are two distinct Y-graphs.
R3: G is a Y-graph if $\alpha(\phi(G))$ is a tree and $\beta(\phi(G))$ is a complete graph, where $\phi(G)$ is obtained from G by the contractions of all the edges of each disjointed Y-subgraphs G_c into a single vertex w adjacent to exactly those vertices that were previously not in G_c and adjacent to at least one vertex in G_c.

The following figure shows a Y-graph with its contraction of a base graph.

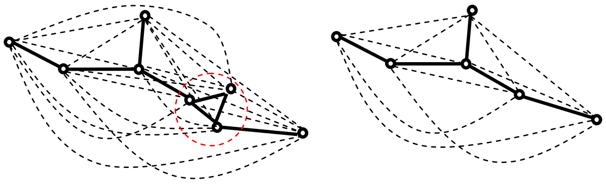


Figure 3. A Y-graph G (left) and $\phi(G)$ (right), where $\alpha(\phi(G))$ is a tree and $\beta(\phi(G))$ is a complete graph.

這種 Y-graph 皆可分析其子圖出現的機率，以下的定理證明此特性。

Theorem 5 [3]: A tree with two or more vertices has at least two leaves.

Given any Y-subgraph (if recognizable), its probability formula can be obtained (shown in the next theorem).

Theorem 6: The probability of a Y-subgraph in a $\Psi(X_n, r, A)$ is computable.

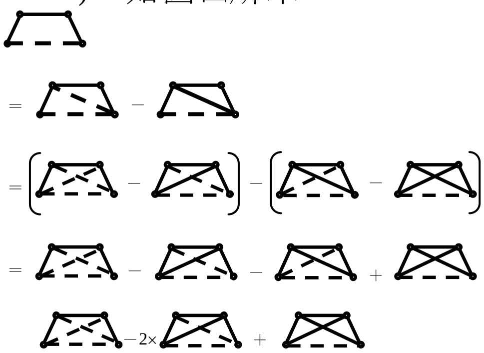
Proof: The probabilities of all base graphs are all computable as shown in Section 4.1. If Pr(G_1) and Pr(G_2) are given, we have Pr(G_1+G_2)=Pr(G_1)+Pr(G_2) and Pr(G_1-G_2)=Pr(G_1)-Pr(G_2). The rest is the case for those Y-subgraphs constructed by applying R3.

Suppose that G is constructed by applying R3. Let S be the size of vertex set of $\phi(G)$ We will prove that Pr(G) is computable by induction on S. When S=1, then G is computable since it is either a single vertex (Pr(G)=1) or a base graph.

Since $\alpha(\phi(G))$ is a tree, $\alpha(\phi(G))$ must contain a leave w by Theorem 5. The removal of w together with the edges incident with it from G results in G^*, which is with S-1 vertices and then computable according to the induction hypothesis. If w is a vertex of G, we have only one solid line and S-1 broken lines incident to w due to the facts that $\alpha(\phi(G))$ is a tree and $\beta(\phi(G))$ is a complete graph; the existence of the unique solid line $e_j=(w, v)$, where v is a vertex in G^*, only depends on whether the distance between w and v is less than r; therefore we have Pr(G)=Pr(G^*)×Pr(E_j). We conclude that G is computable.

Otherwise, w represents a Y-graph G_Y with size less than S. Since every vertex in G^* connects to every vertex in G_Y with broken lines except one solid line $e_j=(x, y)$, where $x(y)$ is a vertex in $G^*(G_Y)$. Similarly, we have Pr(G)=Pr(G^*)×Pr(E_j)×Pr(G_Y); this also implies that G is computable. ■

任意一個圖若要分析其子圖出現的機率目前最有效的方式，爲將此圖化成一些可計算其機率的線性組合(linear combination)。如圖四所示。



圖四. 將圖化成其他圖的線性組合

若干圖無法立即算出一些可計算其機率的線性組合(linear combination)，則可利用一些技巧

來找出其逼近的公示。以下爲一些成功的子圖及其推導。

Theorem 8: For arbitrary four distinct nodes *u*, *v*, *w*, and *x* in a $\Psi(X_n, r, A)$, we have $\Pr(G_S=O=c_4) \le \left(\frac{3\sqrt{3}}{4}\right)\frac{\pi r^6}{|A|^3}$, where $S=\{u, v, w, x\}$.

Proof: Consider the geometric graph c_4 and its circle model (See Figure 5(a) and Figure 5(b)). These four nodes need to be placed properly near to each other in order to form the cycle of length four. Since the longest distance between every two neighboring centers is *r*, the four centers in the circle model must be placed in a convex quadrilateral with the same size length *r* (See Figure 5(c)).

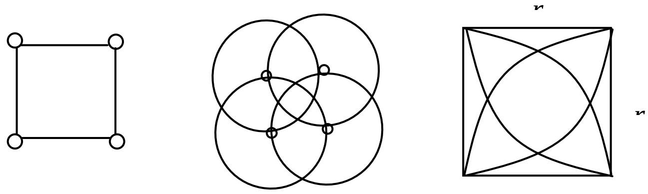


Figure 5. (a) A cycle of length four. (b) Its circle model. (c) The convex quadrilateral in the circle model.

Since the subgraph c_4 consists of a induced path p_2 and another nearby vertex, we have $\Pr(G_S=c_4) \le \Pr(G_{S-\{x\}}=p_2) \times \Pr$(the remaining vertex *x* is near p_2 properly). Because Pr(*x* is near p_2 properly) is the probability of putting the center of *x* in the convex quadrilateral, we have Pr(*x* is near p_2 properly)$\le (r^2/|A|)$. In a sequel, we have $\Pr(G_S=c_4) \le \frac{r^2}{|A|} \times \left(\frac{3\sqrt{3}}{4}\right)\frac{\pi r^4}{|A|^2} = \left(\frac{3\sqrt{3}}{4}\right)\frac{\pi r^6}{|A|^3}$ by Table 1. ■

Theorem 9 For arbitrary four distinct nodes *u*, *v*, *w*, and *x* in a $\Psi(X_n, r, A)$, we have $\Pr(G_S=D=k_4) \le \left(\frac{\pi^2}{2} - \frac{7\sqrt{3}}{8}\pi + \frac{9}{8}\right)\frac{\pi r^6}{|A|^3}$, where $S=\{u, v, w, x\}$.

Proof: The four nodes $\{u, v, w, x\}$ need to be placed sufficiently near to each other in order to form a k_4 (See Figure 6(a)). First, the three nodes $\{u, v, w\}$ must be a triangle c_3. The circle model for c_3 can be presented by intersections of three equal circles (See Figure6 (b)).

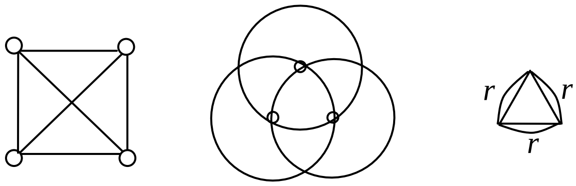


Figure 6. (a) K_4. (b) Its circle model. (c) Reuleaux triangle.

Since the subgraph k_4 consists of a triangle c_3 and another nearby vertex *x*, we have $\Pr(G_S=k_4) \le \Pr(G_{S-\{x\}}=c_3) \times \Pr$(*x* is near c_3 sufficiently). Note that Pr(*x* is near c_3 sufficiently) is the probability of putting the center of *x* in the common intersection of three equal circles; and the largest area of the common intersection, called Reuleaux triangle [30], is $(\frac{\pi-\sqrt{3}}{2})r^2$ (it is easily obtained by summing up the area of a equilateral triangle and three areas of a circular segment with opening angle $\pi/3$). Therefore, we have Pr(*x* is near c_3 sufficiently)$\le (\frac{\pi-\sqrt{3}}{2})r^2/|A|$. In a sequel, we have $\Pr(G_S=k_4)$ $\le \left(\pi - \frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2 \times (\frac{\pi-\sqrt{3}}{2})r^2/|A| = \left(\frac{\pi^2}{2} - \frac{7\sqrt{3}}{8}\pi + \frac{9}{8}\right)\frac{\pi r^6}{|A|^3}$ by Table 1. ■

5. 結果與討論

我們利用這些發展出的推算子圖出現的機率來計算一些無線隨意網路上的理論推

導。包含 The hidden terminal problem, The size of cluster heads, The exposed-terminal problem, The triangle route problem, The recovery mechanism for routing protocols。這些問題在無線隨意網路上的應用及重要性利用下列的表分別說明。以下表的內容利用分析 $p2$ 子圖(圖二)的機率，來計算無線隨意網路出現 The hidden terminal 的數目。

In [12], Khurana *et al*. have shown that if the number of hidden terminal pairs is small and when collisions are unlikely, the RTS/CTS exchange is a waste of bandwidth. On the other hand, if the number of hidden terminal pairs is large, RTS/CTS mechanism helps avoid collision. Moreover, the optimal value for the RTS_Threshold in IEEE 802.11 [12] depends on the number of hidden terminals. Counting the occurrences of p_2 (i.e. $C(p_2)$) helps counting the number of hidden-terminal pairs in the network. We evaluate $C(p_2)$ in the next theorem.

Theorem 10: In a $\Psi(X_n, r, A)$, we have $C(p_2)=3\binom{n}{3}\left(\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2$.

Proof: Since each hidden-terminal pair consists of three distinct labeled vertices, we set S to be the selected three-vertex set. There are $\binom{n}{3}$ different combinations for selecting three from n vertices, and three different settings for labeling one from three as the center of the hidden-terminal pair (*i.e.* the internal node of the induced path with length 2). Therefore, we have the number of hidden-terminal pairs $\binom{n}{3}\times 3\times \Pr(G_S=p_2)=3\binom{n}{3}\left(\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2$ by Table 1. ■

Note that the hidden terminal pairs grow as like $O(n^3r^4)$, where n is the number of mobile nodes and r is the range of power.

以下表的內容利用分析p_2子圖(圖二)的機率，來計算無線隨意網路出現 cluster heads 的數目。

The concept of dominating set in graph theory has been used for hierarchical routing and reducing broadcasting packets [4] in ad hoc networks. Finding the minimum size of dominating set confronts two huge obstacles. First, it is an NP-complete problem, which seems difficult to solve efficiently. Second, it needs overheads to gather global knowledge of the network topology. Therefore, many approaches are proposed to find a dominating set with acceptable size. A simple algorithm for selecting a dominating set has been proposed Wu and Li [23]. Their algorithm selects the center of an induced path with length 2 (that is p_2) as a member of the desired dominating set. We estimate the number of dominating set produced in their algorithm by using the next theorem.

Theorem 11: Given a $\Psi(X_n, r, A)$, the expected dominating set $\gamma(G)\le n\times(1-(1-\left(\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2)^{\binom{n-1}{2}})$.

Proof: For each node (as a center), there are $\binom{n-1}{2}$ possible combinations for other two vertices to form a p_2. The node is not a member of the selected dominating set when every one pair of its neighbors with it does not form any p_2. Therefore, the probability of a node being a member of the selected dominating set is $1-(1-\Pr(p_2))^{\binom{n-1}{2}}=(1-(1-\left(\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2)^{\binom{n-1}{2}})$ (See Table 1). For we have n possible centers, the expected number of dominating set is $n\times(1-(1-\left(\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2)^{\binom{n-1}{2}})$. ■

以下表的內容利用分析M子圖(圖二)的機率，來計算無線隨意網路出現exposed-terminal 的數目。

To derive a tight bound of the number of exposed-terminal sets in a given RGG, we need to compute first the subgraph probability of c_4 (a cycle of length four). The paradigm proposed in Section 4 can be applied to tackle a great deal of subgraphs, but not some types of subgraphs such as cycles. We try to obtain tight bounds for Pr(c_4) in a different way.

Theorem 12: In a $\Psi(X_n, r, A)$, we have $C(M) \geq \left(\frac{3^4-27\sqrt{3}}{4}\right)\binom{n}{4}\frac{\pi r^6}{|A|^3}$.

Proof: Counting the number of exposed-terminal sets is equivalent to counting the number of labeled subgraph M (See Figure 2). There are $\binom{n}{4}$ ways to select four from n elements. Each has $\binom{4}{2}\times 2$ =12 ways in forming the subgraph M (Figure 7).

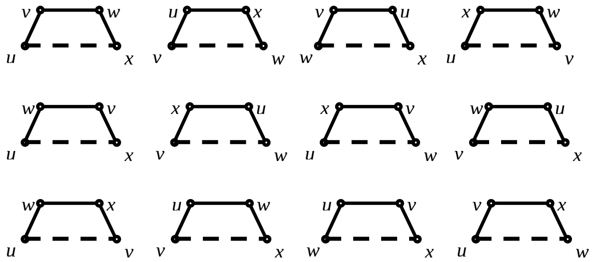


Figure 7. Twelve different ways of labeling M graph.

Note that every graph in the same row contains the same subgraph (cycle of length four). To be accurate, we should avoid such duplicated counting. Therefore the number of exposed-terminal sets is equal to the number of labeled M graphs minus the number of the duplicated cycles (=3(duplicated counting)×3(rows)): $\binom{4}{2}\times 2 \times \binom{n}{4} \times \Pr(G_S=M) - 3\times 3\times \binom{n}{4} \times \Pr(G_S=c_4) = \frac{3^4}{4}\binom{n}{4}\frac{\pi r^6}{|A|^3} - 9\times\binom{n}{4}\times\Pr(G_S=c_4)$

$\geq \frac{3^4}{4}\binom{n}{4}\frac{\pi r^6}{|A|^3} - 9\times\binom{n}{4}\times\left(\frac{3\sqrt{3}}{4}\right)\frac{\pi r^6}{|A|^3} \geq \left(\frac{3^4-27\sqrt{3}}{4}\right)\binom{n}{4}\frac{\pi r^6}{|A|^3}$ (by Table 2 and Theorem 8). ■

以下表的三個定理利用分析c_3(triangle)子圖的機率，來計算無線隨意網路備份路徑出現的機會及修復的可能性。此方法有助於設計無線隨意網路上路由通訊協定上備份路徑之效能評估。首先是估計triangle route出現的個數。

Theorem 13: In a $\Psi(X_n, r, A)$, the expected number of triangle route $\binom{n}{3}\times\left(\pi-\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2$.

Proof: The expected number of triangle routes $C(c_3)$ can be obtained easily since we have computed Pr(c_3)=$\left(\pi-\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2$(See table 1); and there are $\binom{n}{3}$ ways to select three from n elements. Thus, we conclude that $C(c_3)=\binom{n}{3}\times\Pr(c_3)=\binom{n}{3}\times\left(\pi-\frac{3\sqrt{3}}{4}\right)\pi r^4/|A|^2$. ■

其次是估計無線隨意網路備份路徑出現的數目。

Lemma 14 [28]: The expected overlapped area of two properly intersecting circles is $\left(\pi-\frac{3\sqrt{3}}{4}\right)r^2$.

Theorem 15: In a $\Psi(X_n, r, A)$, the expected number of common neighboring nodes of an arbitrary communication link is $(n)\times\left(\pi-\frac{3\sqrt{3}}{4}\right)r^2/|A|$.

Proof: Given two circles with the same radius r in a $\Psi(X_n, r, A)$, the expected overlapped area of two properly intersecting circles is $\left(\pi-\frac{3\sqrt{3}}{4}\right)r^2$ by Lemma 14. Subsequently the probability of a node located in the overlapped area of these two properly intersecting circles is $\left(\pi-\frac{3\sqrt{3}}{4}\right)r^2/|A|$. Finally, the expected number of common neighboring nodes of an arbitrary communication link is $(n)\times\left(\pi-\frac{3\sqrt{3}}{4}\right)r^2/|A|$, where n is the number of randomly deployed nodes and A is the deployed area. ■

最後是估計無線隨意網路備份路徑修復的機率。

Suppose that each communication link has equal probability of independent failure event, denoted by P_F. Also, let P_R denote the probability of that a link successfully recovers from its failure. The following theorem tries to estimate P_R.

Theorem 16: We have $P_R \geq 1-(P_F\times(2-P_F))^{\varphi}$ in a $\Psi(X_n, r, A)$, where $\varphi=(n)\times\left(\pi-\frac{3\sqrt{3}}{4}\right)r^2/|A|$.

Proof: Suppose that there are φ common neighboring nodes of both a and b where (a, b) is a communication link. When link (a, b) fails in a $\Psi(X_n, r, A)$, any of common neighboring nodes of both a and b, say c, can be used to recover from the failure (See Figure 11). In other words, the induced subgraph of nodes a, b, and c form a cycle with length 3 (i.e. c_3); these nodes with node a and b form φ disjoint paths (with length) 2 as backup routes. By Theorem 15, we have $\varphi=(n)\times\left(\pi-\frac{3\sqrt{3}}{4}\right)r^2/|A|$. The probability of failure recovery of each path is thus $P_F\times(1-P_F)+(1-P_F)\times P_F+(P_F)\times(P_F)=P_F(2-P_F)$ because the breakage of any link of the path results its failure recovery. Since there are at least φ disjoint paths, the link recovery fails if all φ disjoint paths fail. Therefore, we have the desired result $P_R \geq 1-(P_F(2-P_F))^{\varphi}$. ■

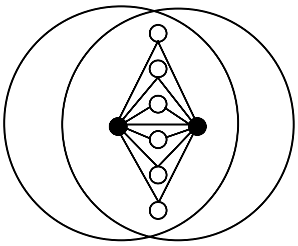


Figure 8. A link (a, b) with common neighboring nodes.

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行政院國家科學委員會補助國內專家學者出席國際學術會議報告

2007 年 8 月 15 日

附件三

報告人姓名	俞征武	服務機構及職稱	中華大學資訊工程系 副教授
會議時間地點	8/1~8/3 Chicago, USA	本會核定補助文號	
會議名稱	(中文)2007 年無線演算法系統應用國際研討會 (英文)International Conference on Wireless Algorithms, Systems and Applications (WASA 2007)		
發表論文題目	(中文)一個部署行動點以連接無線感測網路的新演算法 (英文)Deploying Mobile Nodes to Connect Wireless Sensor Networks Using Novel Algorithms		

報告內容應包括下列各項：

一、參加會議經過

2007 年無線演算法系統應用國際研討會於 8 月 1 日到 8 月 3 日在 USA, Chicago 舉行。7 月 31 日我前往桃園國際機場經 San Fancisco 至 Chicago 時已經是 8 月 1 日。搭乘 CTA(Chicago 之捷運系統)到 Chicago downtown 會議的 venue 處 club quarters hotel 時約早上 8:30am。此時正在舉行第一個 keynote speech 由 David Du (NSF) 談 clean Slate Redesign of Internet for Supporting Services and Applications.下個 session 爲 Routing,正是我十分感興趣的 topics.其中 Prof.Xiao Chen(Texas State University)的 grid 上的 routing 十分有趣，此 session 之後我和 Prof. Xiao Chen 作了短暫的討論.因時差問題,晚上 banquet 我並未參加.

8 月 2 日。Keynote speech 爲 Ophir Frieder(Georgetown University) 談 Search applications on wireless devices. 之後有一篇論文談 underwater sensor network 也是一個相當有趣的問題.

我覺得最有收穫的是接下來的 panel discussion: Holistic algorithmic approaches to sensor networks-Where are we and where do we go from here? Moderator 爲 Taieb Znati (University of Pittsburgh). 有三位教授參加 Wei Zhao (Rensselaer Polytechnic Institute), Cliff Wang (Army Research Office)及交大的 Jason Lin (National Chiao Tung University, IEEE and ACM Fellow).我最欣賞 Prof. Zhao 對 wireless network 的理論基礎之執疑，因這也是我一直感興趣的方向.但 Wei Zhao 的言論也引起很多不同意見.真的存在一個 Holistic algorithmic approaches to sensor networks，需要此研究嗎? Sensor networks 只是一個 distributed networks 或是 interactive algorithms?

下午主辦單位安排 Chicago trolley tour 利用兩小時將 Chicago downtown 逛了一大圈，真是一個美麗的都市.晚餐與 Finance chair Min Song (Old Dominion University)同桌與多位教授相談甚歡.

8 月 3 日。Keynote speech 爲 Jason Lin 談 wireless and mobile all-ip networks. 下午則訪問 Illinois Institute of Technology。

WASA 2007 整個會議因研究方向與我個人相當接近，是一群對 sensor networks 上 algorithm 及理論有興趣的教授組成的，因此這個會議讓我覺得不虛此行.

二、與會心得

我應對 wireless sensor networks 之理論問題作一系列討論.

三、考察參觀活動(無是項活動者省略)

8 月 3 日 visit Illinois Institute of Technology。

8 月 4 日 visit University of Chicago。

8 月 8 日 visit U. C. Berkeley.

四、建議

應鼓勵從事基礎長久深入之基礎研究。

五、攜回資料名稱及內容

WASA proceeding CD

六、其他