

Conservative schemes and degenerate scale problems of null-field method  
for Dirichlet problems of Laplace' s equation

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Abstract

Recently, for circular domains with circular holes, the null-field method (NFM) is proposed by Chen with his groups. In NFM, the fundamental solutions (FS) with the field nodes  $Q$  outside of the solution domains are used in the Green formulas, and the FS are replaced by their series expansions. The explicit algebraic equations of the NFM are first derived in our recent paper \cite {LLHL2010}.

A simple stability analysis is made, and optimal field nodes are explored wherein. The field nodes can also be located on the domain boundary, based on a rigorous proof in \cite {LLHL2010} . However, even for the Dirichlet problem of Laplace' s equation, when the logarithmic capacity (transfinite diameter)  $C_{\Gamma} = 1$ , the solutions may not exist, or not unique if existing, to cause a singularity of the discrete algebraic equations. The problem with  $C_{\Gamma} = 1$  in the BEM is called the degenerate scale problems. Note that the original explicit algebraic equations do not satisfy the conservative law, and may fall into the degenerate scale problem discussed in Chen et al. \cite{CKL2002,CLKC2001,CLCL2002,CS2007}, Christiansen \cite{Ch75} and Tomlinson \cite{TBP96}. In this paper, the new conservative schemes are derived, where an equation between two unknown variables must hold, so that one of them can be removed from the unknowns, to yield the conservative schemes. The conservative schemes always bypass the degenerate scale problem; but it causes a severe instability. A comparative analysis of degenerate scale problems is explored for the conservative schemes and the original equations of the NFM in this paper. A new pseudo-singularity is discovered

that only the minimal singular value  $\sigma_{\min}$  of the discrete matrices is infinitesimal. To restore the good stability of the conservative schemes, the over-determined system and the truncated singular value decomposition (TSVD) are proposed. Moreover, the over-determined system is more advantageous due to simpler algorithms and the slightly better performance in error and stability. More importantly, such numerical techniques can also be used, to deal with all the degenerate scale problems of the original NFM in \cite{CKL2002, CLKC2001, CLCL2002, CS2007} as well as the boundary element method.

Keyword : Null field method, conservative scheme, degenerate scale problem, circular domains, fundamental solutions, Dirichlet problem, pseudo-singularity, overdetermined system, the truncated singular value decomposition