Degenerate scale problems of null-field methods for Dirichlet problems of Laplace's equation

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Abstract

For circular domains with circular holes, the null field method (NFM) is proposed by J.T. Chen and his coresearchers when solving boundary integral equation \cite{CS2009}.
The explicit algebraic equations of the NFM are recently given in \cite{LLHL2010}, and their conservative schemes are proposed in \cite{LLLH2012}.
However, even for the Dirichlet problem of Laplace's equation, when the logarithmic capacity (transfinite diameter) $C_{\Gamma } = 1$, the solutions may not exist, or not being unique if existing, to cause a singularity of the discrete algebraic equations or the original boundary integral equation, and is discussed in Christiansen \cite{Ch75} and many research articles, called the degenerate scale problems. This talk is devoted to a complete and comprehensive analysis on the degenerate scale problem of the NFM for Dirichlet problems. We have found that the conservative schemes can always bypass the degenerate scale problems; though numerically it causes a severe instability. A new pseudo-singularity property is discovered that only the minimal singular value $\sigma_{\min}$ of the discrete matrices is infinitesimal to cause the instability. However, even for the Dirichlet problem of Laplace's equation, there also exist several kinds of pitfalls of field nodes possibly leading to degenerate scale problems, which will be revealed in detail. To remove singularity of discrete matrices and to restore good stability, several effective techniques are proposed, the singular value decomposition (TSVD) is recommended. Numerical experiments are carried out to verify the theoretical analysis made. Based on the analysis and computation, the

trouble of degenerate scale problems when solving by the NFM which have bothered researchers for a long time, see \cite{CLKC2001,CS2007}, can be bypassed.

Keyword：degenerate scale problem, null field methods, Dirichlet problems,